Holographic Optical Elements
for Holographic Stereogram Printers
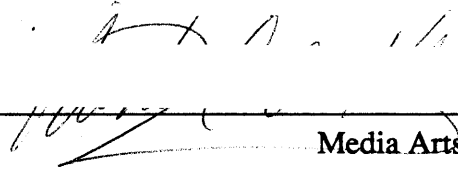
by

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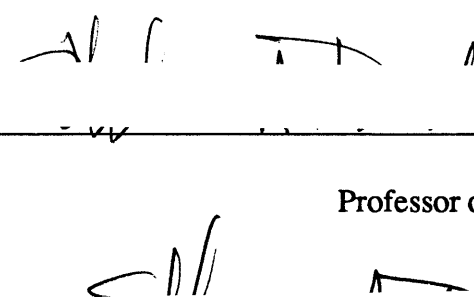Massachusetts Institute of Technology

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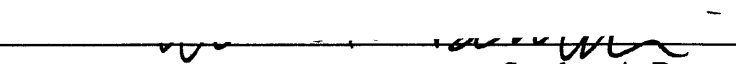10 May 1991

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Holographic Optical Elements for Holographic Stereogram Printers

by

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Submitted to the Media Arts and Sciences Section,
School of Architecture and Planning, on 10 May 1991
in partial fulfillment of the requirements of the degree of
Master of Visual Studies at the Massachusetts Institute of Technology

Abstract

A number of methods for producing holographic optical elements (HOEs) for use in projecting integral images in holographic stereogram printers are presented, with emphasis on application to Ultragram format one-step stereograms. Creation of HOEs to replace standard diffusion screens and enhance stereogram printer imaging properties and efficiency is discussed. Monolithic and integral holographic optical element recording configurations are defined and experimental systems are demonstrated, with emphasis on scalability, ease of manufacture, illumination format, and diffusion property manipulations. Incorporation of rectangular and gaussian HOE-focus intensity distributions is analyzed and HOE-focus relation to stereographic bandlimiting techniques is shown to reduce unwanted image artifacts. Specific examples of application of each HOE to the one-step Ultragram printer are presented and evaluated using a standardized test pattern image. Application of HOE techniques to meter-square format generalized holographic stereogram printers is formulated, with attention given to the specific image distortions that arise from that system.

Thesis Supervisor: Stephen A. Benton
Title: Professor of Media Technology

This thesis is dedicated to my grandfather, who instilled me with the work ethic necessary to "begin it once, and do it".

Acknowledgements

Contents

Chapter 1

Introduction

The development of modern display holography in the 1960's brought with it a promise of revolution in representation of three-dimensional imagery in a two dimensional medium. No longer would a graphic designer have to resort to the rigors of perspective projection to represent a three-dimensional object in a flat medium plane. A scientist wishing to visualize a complex volume of space, be it a physical sample or a computer-modeled equation, would not have to rely on successive two-dimensional perspective views displayed one-by-one on a flat CRT screen to display the three dimensional volume.. The advent of holography would allow the viewer to see inherently three-dimensional objects as three-dimensional images which seemed to occupy a volume of space, but could physically be stowed in a file folder. Perception could be left completely to the viewers, since they could move around the image as if the actual object existed before them. The promise of holography was destined to become a reality as soon as the medium matured.

Twenty-five years, hundreds of scientific papers and thousands of experiments later the realities revealed by the maturation of the medium dissolved or at least postponed this promise of holography. This is not to imply that holographic imaging has not significantly

progressed in that time period. It has just been a progression of detours and shortcuts in order to navigate through 'minor' difficulties which have since been rooted to their places as 'fundamental' characteristics and limitations. The procedure of minimizing the negative effects of the inherent properties of the holographic medium has given rise to such specific practices as horizontal-parallax-only (HPO) imaging, image plane recording geometries, two-step imaging systems, and spatially-multiplexed perspective sampling. These schemes have been devised over the years to reduce both the amount of information as well as the effective distortions that are intrinsic and somewhat debilitating properties of the holographic medium.

The development of holographic stereograms reflects the largest compendium of techniques to both increase the flexibility and decrease the magnitude of distortions that are considered somewhat fundamental to holography as a whole. The earliest experiments in this realm were conducted by R.V. Pole in 1967[23]. Pole was first to introduce a practice involving recording multiple perspectives of an object incoherently, and in white light, and in a later step encoding the perspectives using the coherent laser light interference properties of holography. The resulting hologram would not diffract the wavefronts of light emanating from the surface of the object, as a standard hologram does. Rather, the image produced would be an approximation to the wavefronts that would have emanated from the actual object, were a hologram recorded of it. How closely this approximation matched the actual wavefront determined the fidelity of the imaging system in both Pole's method and subsequent work by DeBitteto, Redman, and McCrickerd and George[8][24][21] . The common thread between all these systems and all other holographic stereogram systems developed since has been the incoherent recording and/or generation of perspectives and the sampling of these perspectives at some frequency which is significantly smaller than the perspective sampling frequency of a standard hologram. The techniques allow for production of holographic images of objects that one could not make an actual hologram of, such as unstable objects, objects that were too large to fit in an optical setup, or objects that never actually existed apart from being 3-D models in the computer.

Work on early stereogram systems concentrated around simplifying the process, in the

hopes that it would one day evolve into a new advertising medium, the ultimate motion picture experience, or perhaps even three-dimensional television. Maverick inventor Lloyd Cross debuted his Multiplex(tm) stereogram, showing that images could be made to be white light viewable in one step, a short time after Benton developed a horizontal parallax only two-step holographic technique with similar characteristics[7][3]. Cognizant of image distortion, but convinced of simple solutions, the pioneers of holographic stereography charted the bounds of the new medium, before zeroing in on the problems of accurate representation. Distortion correction schemes proposed by Benton and later Huff seemingly appeared too late, as interest and hope in stereography waned due to poor image quality. Stereograms were destined to become a static medium with very specific lighting conditions and depth restrictions which would require more effort in both displaying and viewing than originally expected.

Only recently, with the advent of the Alcove hologram and other image processing-intensive systems, and an increase in interest from the medical and design communities for three-dimensional hardcopy, has there been a resurgence in development of holographic stereograms. A good deal of research has been accomplished by the Spatial Imaging Group at MIT in the last five years, developing a tradition of computer-graphic holographic stereography centered around both psychophysical image detail concerns and image processing to undo optical distortions. An exhaustive investigation across the taxonomy of stereography has produced promising specimens which come closer to satisfying the demands of the graphic visualization community. In tandem, the maturing of the computer graphics field, and the evolution of faster computers, has augmented the process of producing the ideal three-dimensional hardcopy display medium.

One research goal is to produce a hardcopy peripheral printer, capable of generating holographic stereogram hardcopy output with little or no user intervention. Proceeding from a tradition of photographic and optical physics combined with computer graphics and image processing, the challenge in developing such a device is significant. Recent advances in image predistortion and rendering techniques, as well as materials improvements have sustained the

possibility of producing such a stereogram printer. The process of streamlining the system includes consideration of light efficiency, automated mechanization, and development of a sound, stable combination of optical and image processing elements.

This thesis addresses two goals in regards to the streamlining process: first, the application of holographic techniques, and, somewhat recursively, stereographic techniques to the simplification and improvement of the optical system of the stereogram printer; and second, the complete analysis of established printing techniques and improvements thereof which serve to improve the cosmetic and accuracy aspects of stereographic imaging. The marriage of the recently-introduced Ultragram generalized stereogram with hybrid holographic optical elements promises to be a precursor for a prototype three-dimensional hardcopy peripheral printer. An added benefit is scalability, which indeed merits its own section in this document as a significantly challenging issue. This is not a promise of answers to all the image quality issues, but rather an investigation in hopes of fully understanding the limitations and physical realities inherent in the holographic stereogram system.

Chapter 2

The Ultragram

When the limitations of traditional horizontal-parallax-only stereogram models are analyzed, their characteristics can be broken down into a number of general categories. Many of these characteristics are of concern in developing a generalized format for holographic stereograms. Some addressable issues include:

- Viewangle.

    We seek a stereogram format with the widest angle of view possible to insure a more realistic representation of the image volume.

- Illumination format.

    Due to spatial limitations, a choice between reflection and transmission schemes must be made, and planned illumination distance limitations must be considered. Also the question of direct versus phase-conjugate illumination must be addressed.

- Component image adaptability

    The system should be able to produce three-dimensional hardcopy using existing graphics packages and perspective images and data sets collected using the standard means

available.

- Printer simplicity.

  The stereogram printer used should require neither expensive nor image-degrading optical elements.

- Scaling potential.

  The system should be scalable to allow for very large image volumes to be represented.

The Ultragram was developed as a general package which would allow flexibility in all of these format issues without compromising image quality in the process. By adapting a scheme in which the viewer's position, integral exposure position, shear plane and object position are elastic variables, the Ultragram allows for a variety of image representations to be presented to the viewer.

Image Processing

Much of the existing holographic stereogram technology is the legacy of groundbreaking work done by D. J. DeBitetto in the late 60's and early 70's. DeBitetto incorporated a step-and-repeat process in recording the integral exposures that make up the stereogram plate. The resulting spatially-multiplexed perspective views could then be transferred in a second holographic step to create an image whose discrete perspectives were angularly multiplexed. The important factor in the DeBitetto technique is the placement of the integral slit exposures at the eye, such that the viewer's pupil overlaps the slit pupil in order to see the correct image view.

Today's standard stereogram techniques in the DeBitetto tradition rely on a two-step process for white-light viewability. In the first step the perspective images are projected onto an isotropically diffusing screen, and an integral exposure is made of each one. A second step is required to make the object volume straddle the image plane, in order to decrease the effects

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of chromatic dispersion. The second step also serves to place the integral exposure plane in space at the viewer's eyes, insuring that the image plane is the plane of maximum angular multiplexing. The DeBitetto stereogram method is successful and has thrived for a number of reasons. First, because the integral slits are separated from the plane of the plate in the white light transfer, source size blur and horizontal chromatic dispersion serve to expand the integral width, and thus overlap neighboring integral exit pupils slightly on top of each other. This effect serves to remove unwanted "picket fence" effects caused by the coarse sampling of perspectives. Another characteristic associated with the DeBitetto transfer is the default placement of the viewzone placement at the plane of the viewer's eyes. This results in a physical limitation of the relative placement of the master and transfer planes during recording. A variation in the distance between the two planes which is not in proportion with the scaling of the inter-sample distance results in a depth distortion in the image. Thus, in order for the viewers to see an undistorted image, they must place their eyes in the image of the integral plane, and the recording geometry of the stereogram must be in correspondence with the recording geometry of the original perspective images. In addition, because the placement of the master and transfer plates determines the windows through which the image will be seen, the larger the distance between these two planes, the narrower will be the view angle of the final image.

Perhaps the most significant benefit of the DeBitetto system is the direct method by which the perspective images are recorded or generated. The camera model for the images is simple because the geometry between the viewer and the hologram is fixed. However, it is this restricted geometry of the DeBitetto stereogram method that is at the root of it's limitations of viewangle, image distortion and inefficiency. An analysis of the system shows that by a simple image processing step, the limitations governing the DeBitetto model can be relaxed and manipulated to improve the characteristics of the final output. This manipulation gives rise to what has become known as the Ultragram system.

The kernel of the Ultragram system is a graphics postprocessing step that accepts the DeBitetto-style perspective views of the subject as input, and recombines those views to produce

a new set of predistorted images that are then used as input for the stereogram printer. Dubbed "slice-and-dice", because of its image recombinatorial nature, the parameters of this program can be adjusted to predistort the input images to compensate for distortions caused by changing the relative diffusion-screen-to-integral-plane distance.

A detailed description of the Ultragram system is included in Halle[14]. A simple ray-tracing analysis of the generalized stereographic imaging system is presented here to acclimate the reader to the issues that concern these experiments.

Referring to Figure 2.1, we see a volume of information represented, which we would like to view from a number of different points, as indicated by the viewpoint line. To simplify our analysis, we choose only to view these points from a fixed vertical position, implying that only horizontal parallax information will be gathered. If a perspective image is generated or recorded from a given viewpoint position and displayed in a pixellated format on a projection screen, the columns of image information in that projection can be mapped to a different integral perspective collection space. For instance, in Figure 2.1a, column #8, the middle column in the image displayed from viewpoint #6 contains the same information as column #6 of integral exposure #8 in Figure 2.1b. A similar mapping can be found for every point in the image volume. The images on the projection plane represent those of an anamorphic camera, in which the vertical focus has not changed from the DeBitetto case, but the horizontal focus has moved much closer to the projection screen. This is the essence of the Ultragram predistortion: anamorphic image processing to allow positioning of the projection screen and integral exposures anywhere in the shear plane-viewer continuum. For any position, the mapping can be found and the graphics predistorted to compensate for the change in screen-integral plane spacing and orientation.

The Ultragram format is important for a number of reasons. The flexibility in positioning the integral exposure plane allows for the possibility of a much larger angle of view, because the system is no longer dependent on placing the image of the perspective slits at the viewer's eye as in the standard DeBitetto stereogram. Also, the images may be predistorted such that direct
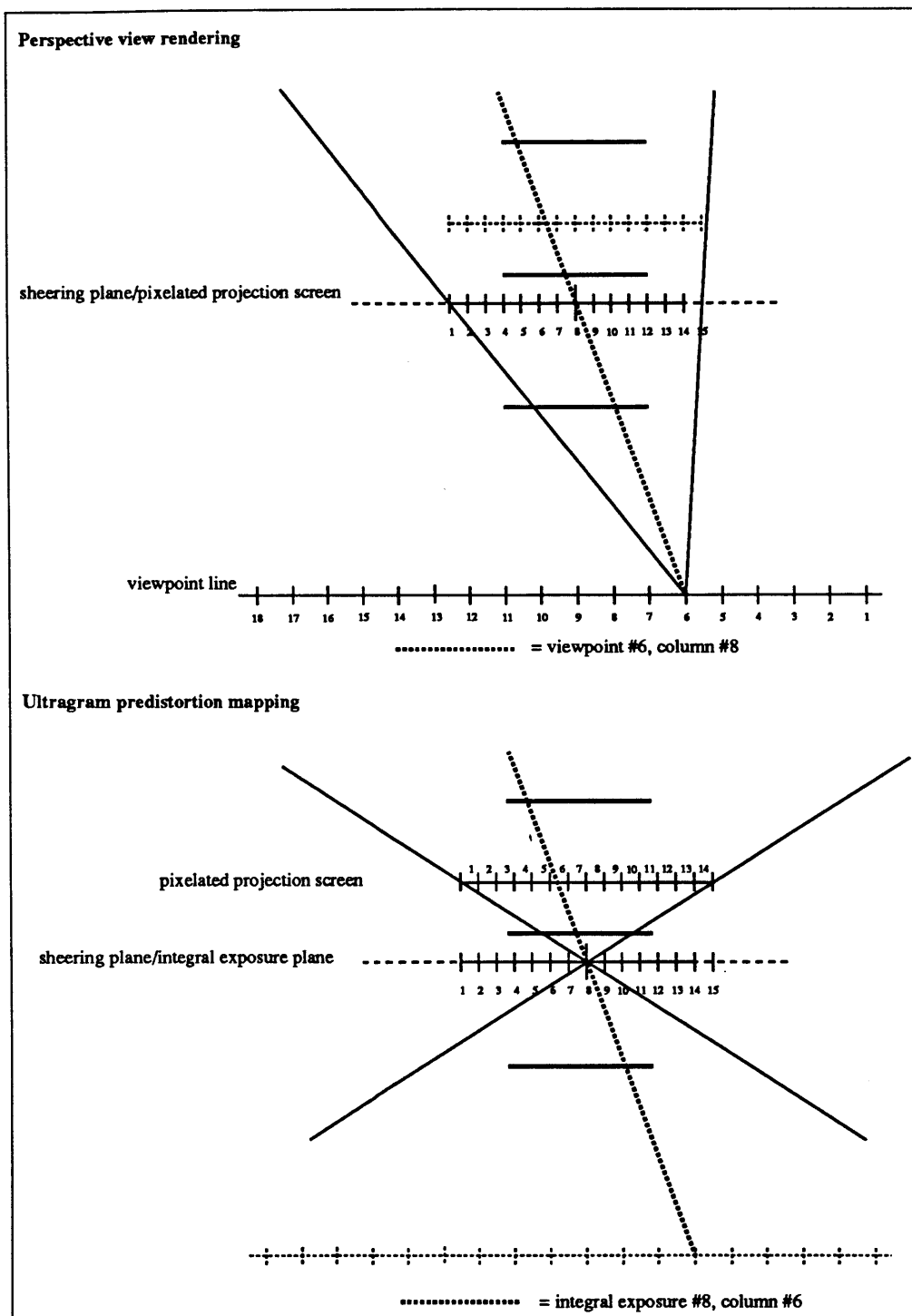
Figure 2.1: Mapping of normal viewpoint perspective information to Ultragram anamorphic space. The three bold horizontal lines represent an image consisting of three square planar objects
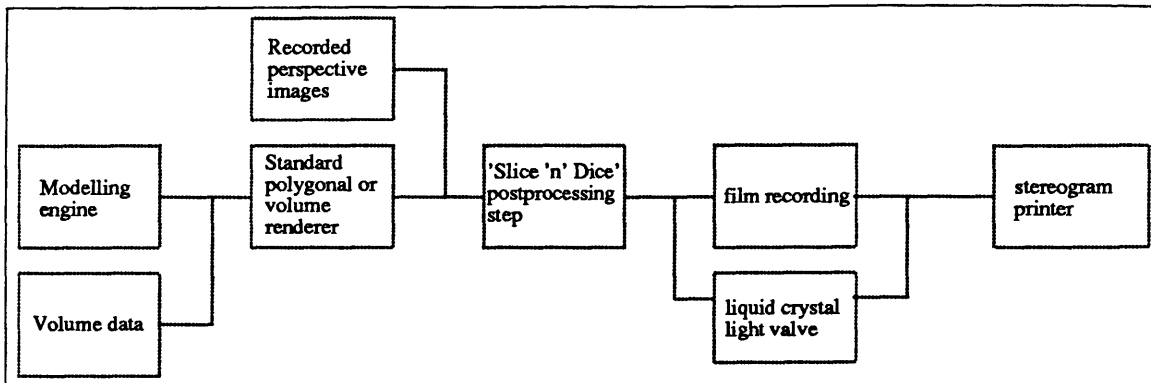
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Figure 2.2: The imaging pipeline for the Ultragram holographic stereogram system. Data from a computer graphics modeller or volume data from medical or other sources is rendered to produce a number of discrete perspective views of the subject from the point of view of a camera on a track. Similarly, perspective images recorded in just this manner can also be input into the "Slice'n'Dice" predistortion correction system. Finally, the predistorted images can be downloaded to a film recorder or a light valve for input into the stereogram printer itself.

illumination rather than phase-conjugation may be used for the image-plane transfers. Finally, the integral slits may be placed anywhere in the continuum, even on the image-plane, enabling the possibility of one-step stereogram imaging. The flexibility of the Ultragram generalized stereogram makes it possible to produce images in a variety of formats depending on the needs of the user. A schematic of the Ultragram image pipeline is diagrammed in Figure 2.2.

Format Issues

Another decision to be considered in the general stereogram system is the question of reflection versus transmission format. Benton's improvements on DeBitetto's system called for the perspective image projection screen to be placed at the achromatic angle with respect to the master plate, so as to minimize chromatic dispersion effects[4]. Because the input images for the Ultragram are predistorted according to the distance between diffusion screen and integral plane, however, it becomes difficult to incorporate achromatic angle placement of the screen in

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order to compensate for chromatic dispersion. Although the postprocessing presumably could be adapted to change according to vertical position and thus make achromatic angle exposure possible, this computation would be very costly, and would be detrimental to the streamlining of the Ultragram system. Consequently, production of an Ultragram in transmission format, to be replayed in broadband light, implies some amount of chromatic dispersion. If a reflection format is used, the chromatic dispersion problem is minimized, due to the narrow bandwidth reflectance of the gratings formed.

Unlike the DeBitetto case, Ultragram predistortion techniques allow for placement of the perceived object volume anywhere in the viewer-projection screen line. If chromatic dispersion were not an issue, Ultragraphic predistortion would be sufficient to produce stereograms in one step. Unfortunately, due to the existence of chromatic dispersion, even reflection format stereograms produced in this manner suffer color blurring that lowers the effective resolution of the image. Figure 2.3 details the vertical chromatic dispersion in a reflection stereogram with a bandwidth of 17 nanometers. In that case, the playout of a point imaged on the center axis is calculated to span a range of $1.3°$. For the case when the diffusion screen is separated from the hologram plane by a distance of 100 mm, the vertical chromatic dispersion spreads a 1 mm point over 2.27 mm, which, though not ideal, is tolerable. However, in another case, where the diffusion screen is moved to a distance (d_{screen}) of 500 mm from the hologram plane, a 1 mm point is spread into a 11.35 mm wide wavelength continuum, a characteristic that degrades the resolution of the final image significantly. The difficulty arises because the diffusion screen creates a local focus for a given point in space that is then recorded in the integral stereogram exposure. Upon playback, that point is reproduced in space at exactly its original position with the same wavelength used to record it. Multiple wavelengths also reproduce images of the point, however, due to the finite bandwidth of the system, and they are displaced in space vertically as described by standard paraxial raytracing calculations. A second step that images these points to the image-plane of the hologram eliminates the chromatic dispersion problem by localizing the point foci to the image-plane. The second hologram can then be viewed in

recording

d_{screen}

diffusion screen

image point

θ_{ref}

hologram

$\theta_{ref} = 60°$

$\theta_{obj} = 0°$

illumination

broadband source

diffusion screen image

λ_1

image point wavelength continuum

λ_2

θ_{ill}

hologram

$\theta_{ill} = \theta_{ref} = 60°$

$\lambda_1 = 647 \, nm$

$\lambda_2 = 630 \, nm$

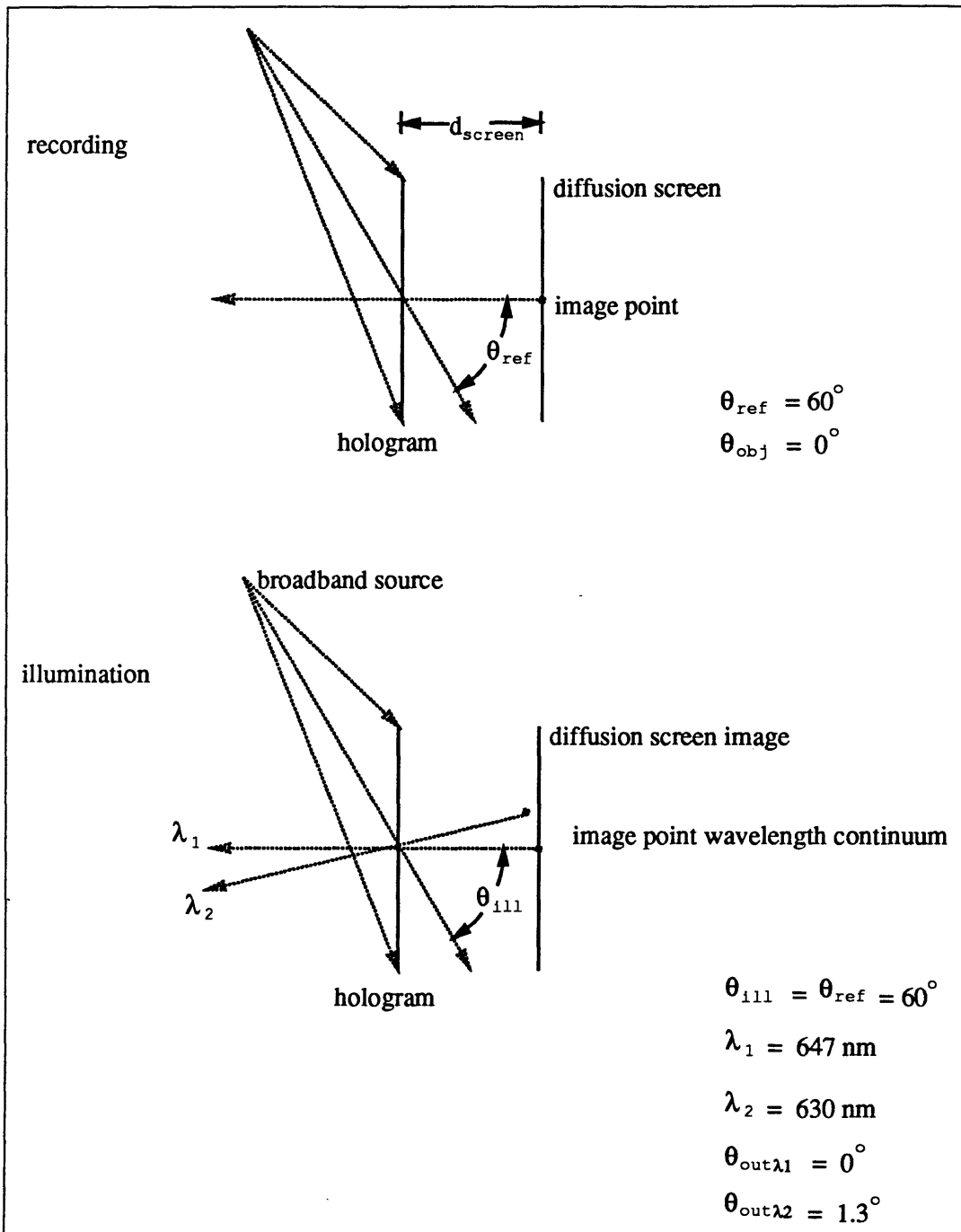$\theta_{out\lambda1} = 0°$

$\theta_{out\lambda2} = 1.3°$

Figure 2.3: Vertical chromatic dispersion quantified for a 17 nm bandwidth hologram imaging a point on a isotropic diffusion screen.

white light without significant image degradation due to chromatic dispersion. In this sense, stereogram printers using isotropic diffusion screens are in a way intrinsically linked to the two-step approach by virtue of the location of the vertical focus plane.

The two-step Ultragram method is very effective at producing images that have a very wide vertical and horizontal view angle, retain excellent resolution, and have few noise artifacts as compared with the standard DeBitetto type stereograms. As two-step images, their primary detriment is the lack of achromatic angle considerations, making it difficult to produce multi-color images. This same characteristic implies that reflection mode is the preferred format for the two-step Ultragram, due to the narrow band playback that it offers.

Because Ultragram predistortion will allow placement of the object volume anywhere with respect to the integral slits and the diffusion screen it is the vertical diffusion problem that prohibits the Ultragram format from producing one-step image-plane-bisecting image volumes. Initial experiments in one-step Ultragram images were made with an isotropically diffusing screen (just as in the master recording step in the two-step system) in transmission meter-square format. The final image exhibited severe chromatic dispersion when illuminated with white light, so a narrow-band arc source was used to minimize this dispersion for the display. The image was produced to experimentally demonstrate the concept of one-step image-plane Ultragraphic displays. Figure 2.4 shows the final image as displayed with the arc lamp.

Inherent in the geometrical relationship between the projection screen and the integral slit is the concept of angular multiplexing, that allows the images to be perceived as three-dimensional in the first place. Because the holographic stereogram is a horizontal-parallax-only system, the spatial separation between the projection screen and the integral plane is only crucial in the horizontal dimension. Thus, it follows that if the diffusion screen can be separated into vertical and horizontal screen elements, and the vertical element can be moved to the integral plane, the chromatic dispersion in that dimension could be removed while angular multiplexing could still exist horizontally. A similar concept is noted in the Multiplex (tm) system, in
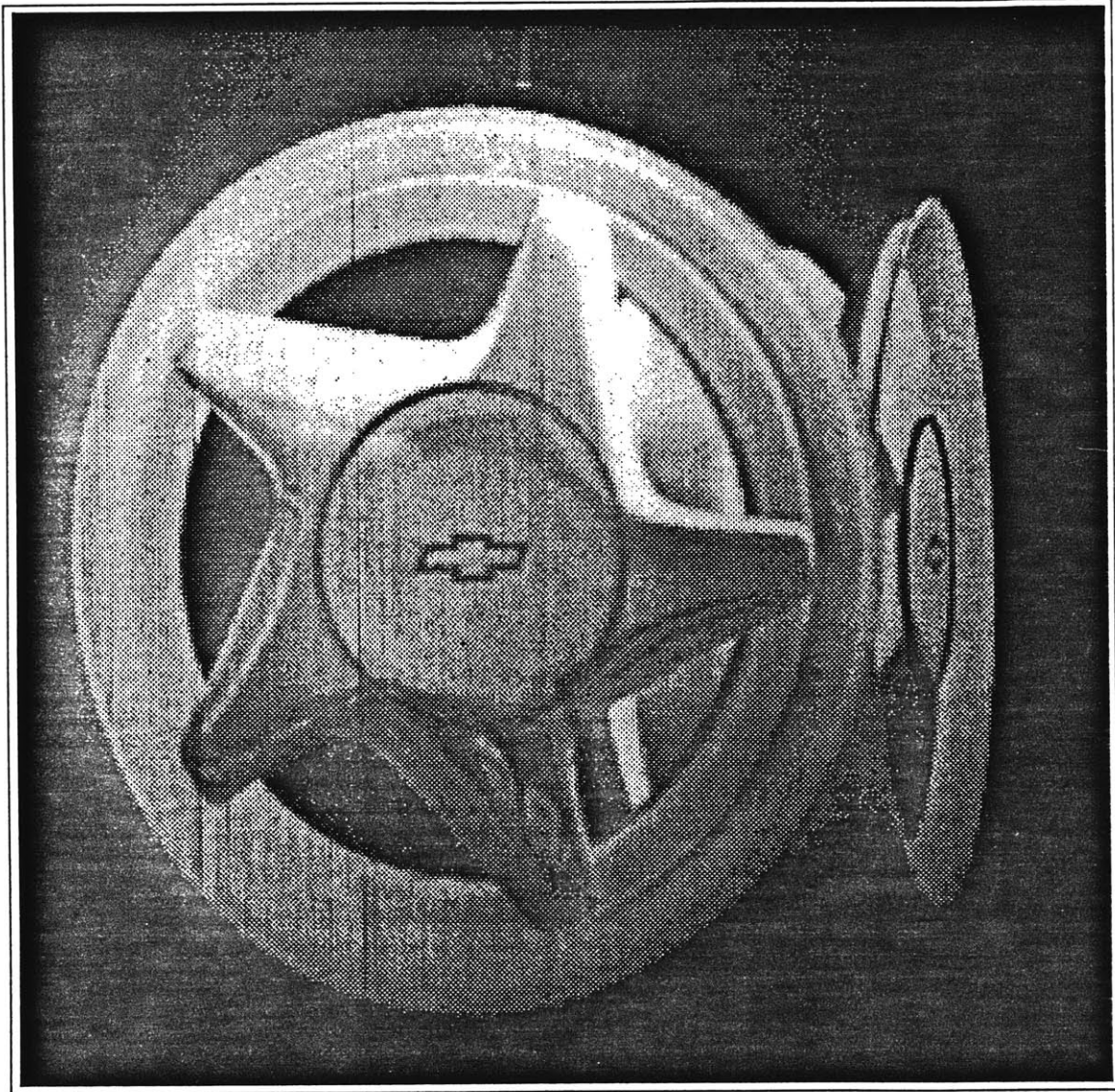
Figure 2.4: Meter-square one-step Ultragram made using an isotropically-diffusing projection screen and illuminated with a filtered mercury arc lamp source.

which angular multiplexing is ensured via horizontal compression of information projected onto the aperture of a large cylindrical lens into the integral slit. The vertical focus is placed at the view distance, far from the integral exposure plane, rendering this an astigmatic system. Vertical diffusion is then caused by chromatic dispersion from the hologram plane of points projected onto the image plane with infinite or very distant focus. The Multiplex (tm) hologram suffers from the fact that its input images are not predistorted so that the image volume cannot approach the hologram surface. Even if the component images were predistorted correctly, however, horizontal chromatic dispersion combined with the broadband illumination of the Multiplex (tm) playout will make the image distort and change size as the viewer moves his/her head vertically. Reconfiguration of the system to reflection mode narrows the reconstruction bandwidth, but vertical dispersion almost completely disappears, allowing only the part of the image in direct line with the viewer's eye to be visible. Huff corrected this problem by inserting a periodic diffuse, (in this case a lenticular screen), in front of the integral focus during recording. This diffused the light sufficiently to provide a useful vertical angle of view. For the horizontal focus, Huff also used a lenticular screen in place of the large-aperture cylindrical lens in some experiments, although he reported poor image quality due to the periodicity of the screen.[16]

It has thus been established that chromatic dispersion plays a very important role in determining the most ideal exposure geometry for the Ultragram system. This is especially the case in a one-step geometry, in which anamorphic techniques are required to prevent significant distortion or image blur.

Component Image Projection Systems

Figure 2.5 shows a comparison between the diffusion properties of several of the stereogram systems described here. Huff's system suffers from the periodicity of the diffusion screen, unlike the Benton and DeBitetto approaches, which provide randomized although isotropic
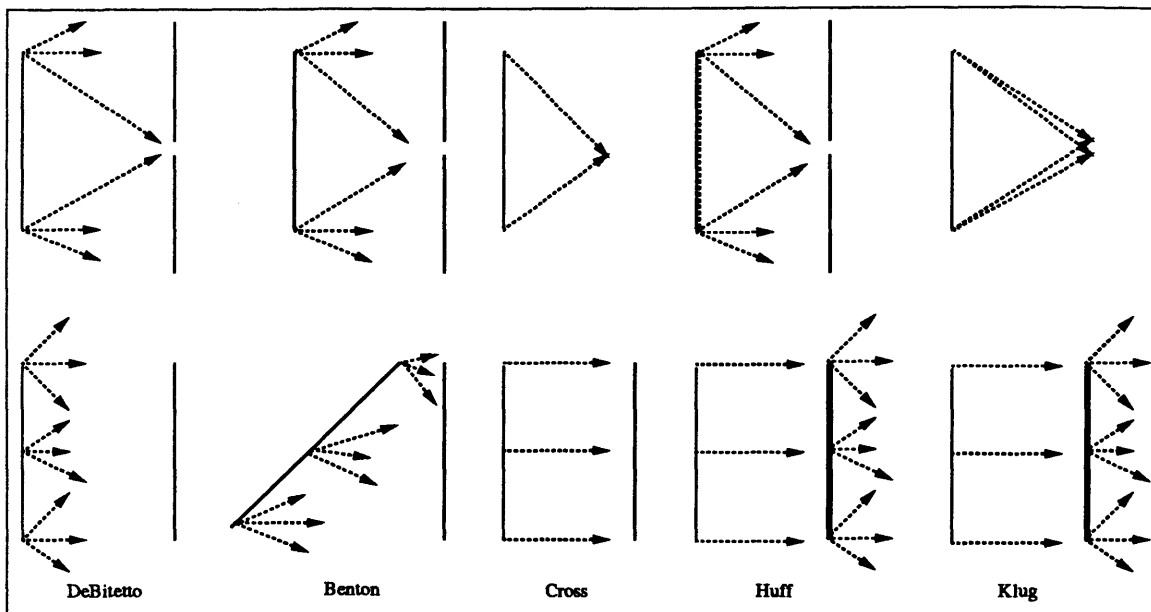
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Figure 2.5: A comparison of the diffusion characteristics of some holographic stereogram printer projection systems.

diffusion vertically. All three systems suffer from inefficiency in directing the light to the exposure-defining slit, as opposed to Cross' original design that focuses all of the image light to an extremely narrow, if aberrated focus. It seems to the author that a hybrid of all the systems could provide reasonable characteristics for an improved one-step system. Thus, the system proposed here would use a specialized screen to converge the light to a finite-width focus or series of discrete displaced foci that would retain horizontal diffusion from the projection-screen plane, while conserving light projected upon that plane. It would also disrupt the periodicity of the light so as to retain image quality without interference effects due to the screen. In addition, the proposed, and hereafter described screen would correctly abut neighboring slit or integral exposures, unlike the Cross system, which suffers from aberrations due to slit-plane exposure discontiguity. While the DeBitetto, Benton and Huff systems require a physical slit to delimit exposure boundaries, the Cross method and that proposed by the author do not, thus simplifying the object beam projection train by avoiding optical effects of mechanical intrusions into the

exposing beam path. Finally, using Ultragram predistortion methods, this system would display an image volume that could straddle the hologram surface in one holographic step, and render the image white-light viewable in reflection mode.

Chapter 3

Holographic Optical Element Configurations

This chapter will discuss the implementation of cylindrical lens and isotropic and anisotropic preferred-direction diffusing holographic optical elements. Due to the specific optical properties we seek to produce in the one-step Ultragram format, the term 'anisotropic' implies diffusion in only one dimension, while 'isotropic' refers (perhaps simplistically) to diffusion in two dimensions that are mutually orthogonal. Thus, we will hereafter refer to the experimental elements as CYL LENS HOEs or 1-D or 2-D PDD HOEs (1-Dimensional or 2-Dimensional Preferred-Direction-Diffusing Holographic Optical Elements).

Two different approaches to synthesizing both CYL LENS HOEs and PDD HOEs were analyzed. The standard, perhaps most obvious approach involves recording a hologram of a narrow strip cylindrically or anisotropically diverging element that has been illuminated as diagrammed in Figure 3.1.

In this format, in which the entire HOE aperture is formed in a single exposure step, is termed *monolithic* due to the uniform nature of the exposure across the aperture. In a second
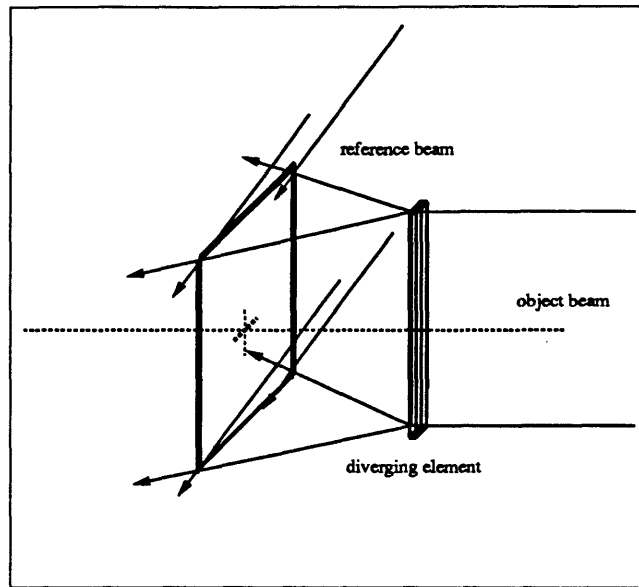
Figure 3.1: Monolithic HOE recording.

approach, diagrammed in Figure 3.2, the HOE aperture is subdivided into arbitrarily narrow strips, and temporally multiplexed exposures of a much smaller diffusive element are made in the strips one-by-one until the entire aperture has been exposed. This method is termed *integral* due to the segmented nature of the HOE exposure. The two types of HOEs can be compared and contrasted in much the same way as can holograms and holographic stereograms: the integral method gives a segmented approximation to the wavefront that the monolithic method produces. Both types of HOEs have advantages and disadvantages that will be discussed in detail.

Integral HOE recording

strip reference beam

diverging element

Integral HOE playback

illumination beam

integral line focus
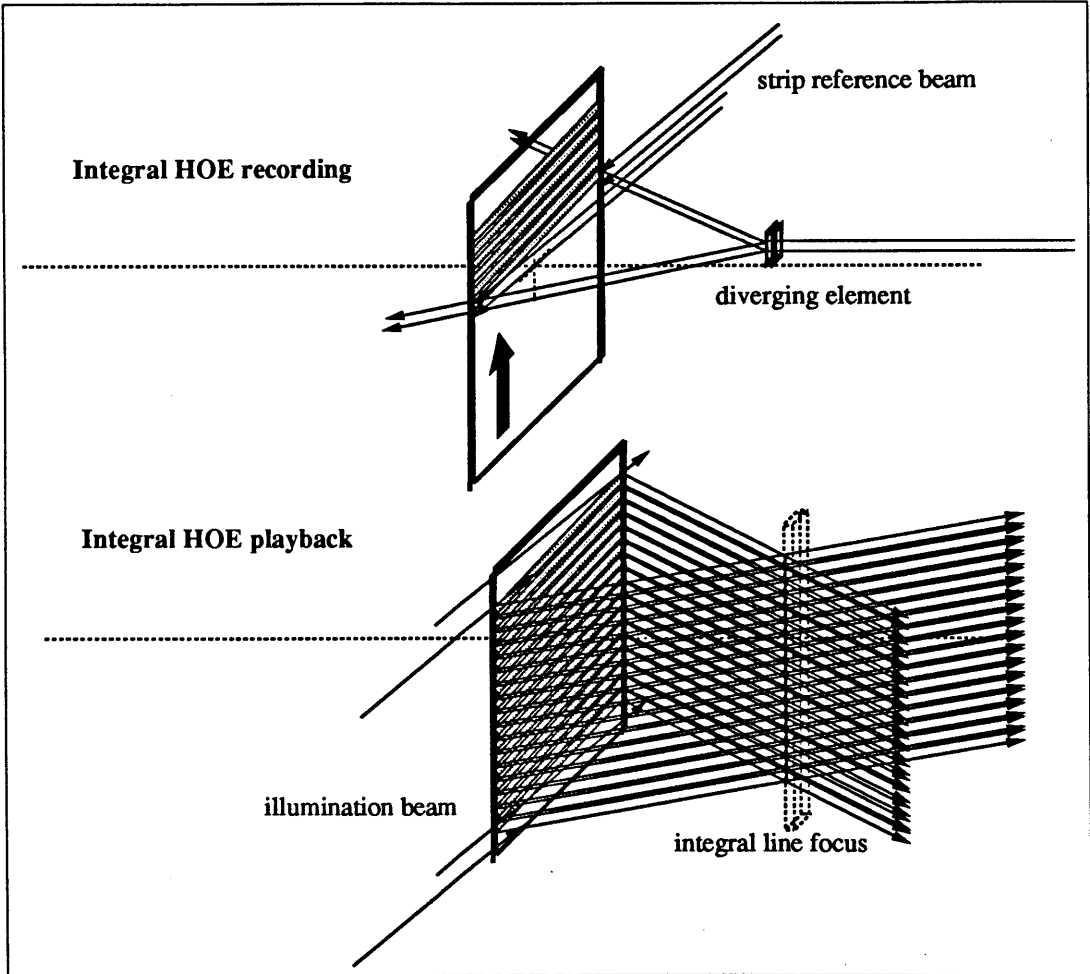
Figure 3.2: Integral HOE recording and playback.
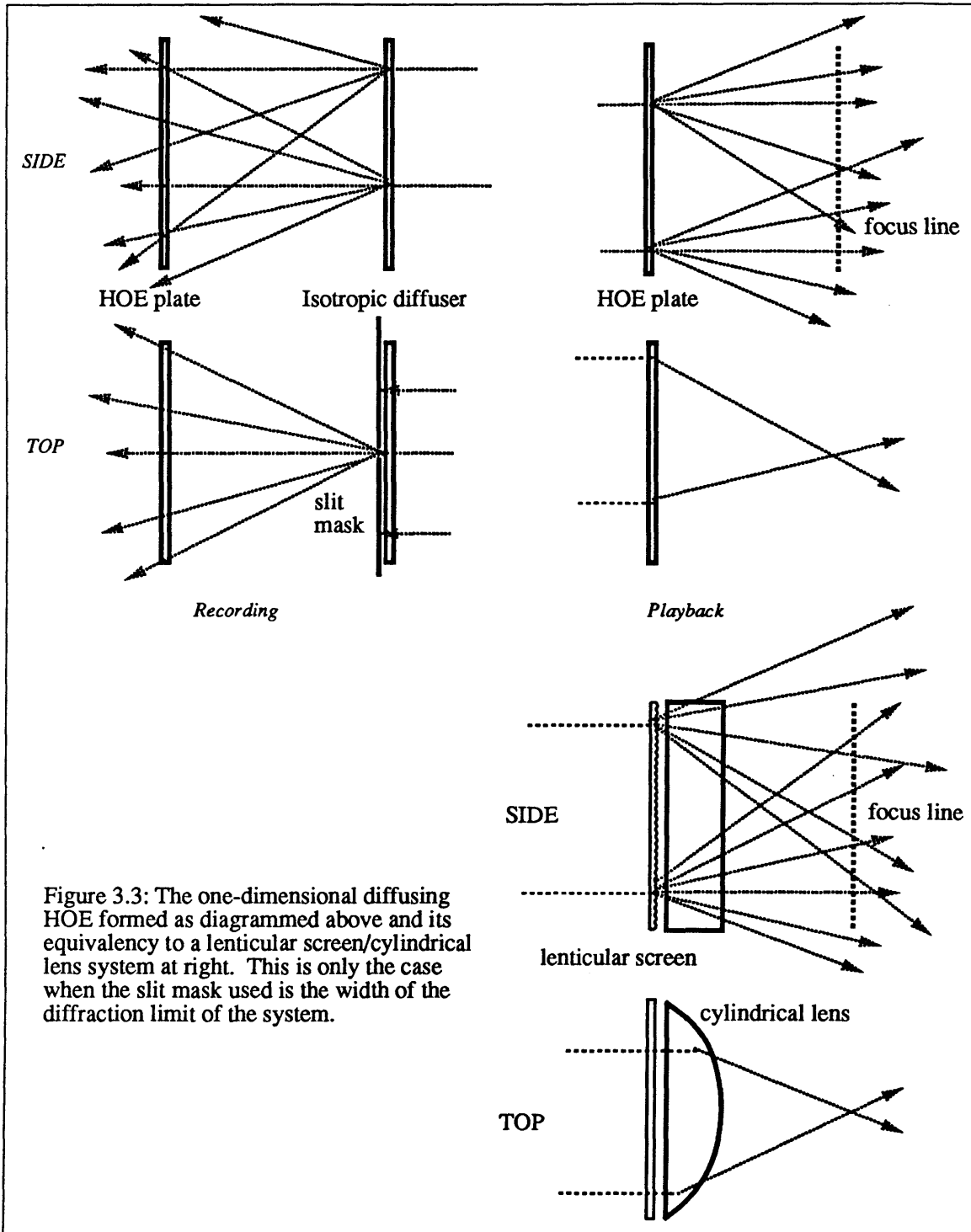
Monolithic HOEs

One-step Recording

A monolithic HOE is made by recording a hologram of a strip diverging element and an off-axis reference beam. The diverging element, hereafter referred to as an object-beam-diverging element (ODE) is an optical device with varied diffusion and divergence properties, depending on the particular case. In the case of a PDD HOE, the ODE is a strip diffuser with either isotropic or anisotropic diffusing characteristics, depending on the application. Previous experimentation by Benton on this type of HOE involved using an apertured, coated commercial projection screen with isotropic diffusion properties akin to those of ground glass. In this system, light impinging on the diffuser through the slit mask is distributed randomly onto the holographic plate beyond and interfered with a collimated off-axis reference beam. When the hologram is illuminated with the phase conjugate of the reference beam, a real image of the diffusing slit is formed in space. When the real image is placed on the holographic stereogram plane (in the context of a stereogram printer), and the illumination beam is modulated with image information, a conventional DeBitetto-type integral exposure can be made. The resulting stereogram has the same diffusive characteristics as the DeBitetto-type stereogram, the HOE serving simply as a light-conservation device by only directing light toward the slit as opposed to equally dispersing it in all directions. Benton combines this type of HOE with a off-axis collimating element and louver light control film which allows the images to be projected onto a plane orthogonal to the projection beam[4]. The louver film prevents the zero order beam from impinging on the stereogram exposure. Huff and Fusek describe a similar monolithic HOE for a multiplex imaging system, using a long cylindrical lens as their object beam diverging device, rather than a diffuser[10].

It may be helpful at this point to clarify the differences and similarities between cylindrical lenses and preferred-direction diffusing HOEs. In the cylindrical lens case, when the HOE is illuminated with the phase conjugate collimated beam, the diffracted light converges to a

nearly-diffraction-limited focus in one direction, while retaining its collimated status in the the other. Thus, there is a one-to-one mapping of a column on the HOE (orthogonal to the focus line) to a point on the focus, and vice-versa. A PDD HOE, however, while converging the light to a similar focus in the same dimension as the cylindrical lens, may diffuse the light such that that focus is no longer diffraction limited, or such that one-column on the HOE maps to many points on the focus or both. In a simple example, diagrammed in Figure 3, we can model a PDD HOE made by exposing the holographic plate to a ground glass screen masked with an very narrow slit as a cylindrical lens sandwiched with a one dimensional diffuser that diffuses the light in a direction parallel to the slit. In both cases, a single column on the HOE maps to an infinite number of points on the focus. This illustrates that, in many cases, the diffusive and the optical approaches to producing stereograms are not mutually exclusive. In the above example, the main difference between the two systems is the width of the slit used to record the PDD HOE. In the limit as the slit width approaches the diffraction limit, the HOE "becomes" a cylindrical lens in the vertical dimension.

While relatively simple to produce, the properties of a one-step PDD HOE will strongly depend on the radiative characteristics of the diffusive element used to make it. Thus, because the radiated field intensity of the diffusion slit tends to peak in the center and fall off sharply toward the edges of the hologram plane, any image that is subsequently projected on the HOE will have the same characteristics in the final stereogram exposure. In other words, light intensity contributions to the integral slit will be much higher from the center of the illumination beam than from the edges. This problem is difficult to remedy in a single exposure step because it is caused by the basic dispersive properties of the diffuser. Typical coated rear-projection screen diffusers exhibit an intensity dropoff of approximately 8 to 1 over an angle of 90 degrees from the center to the extreme edges of the radiated field at a distance of 30 centimeters from the diffuser. An additional problem with the one-step approach is the difficulty in producing a 1-D PDD HOE. The original screen used to produce this type of HOE must have anisotropic diffusion properties, and a lenticular screen is commonly used.

SIDE

HOE plate   Isotropic diffuser   HOE plate   focus line

TOP

slit
mask

Recording   Playback

SIDE   focus line

lenticular screen

cylindrical lens

TOP

Figure 3.3: The one-dimensional diffusing HOE formed as diagrammed above and its equivalency to a lenticular screen/cylindrical lens system at right. This is only the case when the slit mask used is the width of the diffraction limit of the system.

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Unfortunately, when only a small portion of a lenticular screen is illuminated through a slit mask placed parallel to the lenticules, only a small number of lenticules (typically no more than 4 or 5) diffuse the light. When the HOE is illuminated with the phase conjugate of the reference beam, the slit image that is formed then consists of the discrete foci of each lenticule, as opposed to a continuous randomized diffusive distribution, due to the periodicity of beams emerging from the lenticules. These weaknesses of one-step monolithic HOEs have motivated investigation into two-step approaches to producing PDD HOEs.

Two-step Recording

The two-step PDD HOE is produced by first recording a narrow off-axis strip hologram (H1) of a diffusion screen. This hologram is then illuminated with the phase-conjugate of the reference beam and a second hologram (H2) is recorded with the radiated field and a collimated reference beam. The H2 is finally illuminated with its reference phase conjugate to form a real image of the strip H1. There are two main benefits to producing the HOE with this two-step approach. First, by properly masking the intensity of the light illuminating the diffusion screen during recording of the H1, the screen can be made to be equiluminant across its surface from any point of view along the H1 strip. Thus, when the H1 is phase-conjugate-illuminated, the field that it projects onto the H2 plane will be equal in intensity at all points, ideally representing a perfectly Lambertian diffuser in at least one dimension. A second benefit to the two-step monolithic HOE scheme is the disruption of periodicity in the one-dimensional diffuser case. When the strip H1 is recorded of a full-aperture lenticular screen, then illuminated with the phase-conjugate, a real image of the lenticular screen is produced on the plane of the H2, in addition to a significant noise factor caused by object self-interference and screen imperfections. When the H2 is illuminated, the radiated field at the real image of the slit H1 resembles that which a very high-pitch lenticular screen illuminated with a beam with a randomized phase front would produce at the same distance. Thus, little or no periodicity is observed within the slit focus, although one-dimensional diffusive characteristics are retained.

Experimentation with both 2-D and 1-D (iso- and anisotropic) PDD HOEs prove that this is indeed a much more favorable approach than the one-step results, provided that proper intensity filtering occurs. Intensity filtration was obtained by first measuring and plotting the intensity distribution characteristics of the coated diffusion screen. It was then determined that an inverse curve could be closely approximated by a one-dimensional Gaussian transmittance function that would reduce the intensity of the light impinging on the center of the screen and not effect the intensity on the edges. The characteristic radiative curve and the inverse Gaussian intensity filter are plotted together in Figure 3.4. The inevitable loss of diffraction
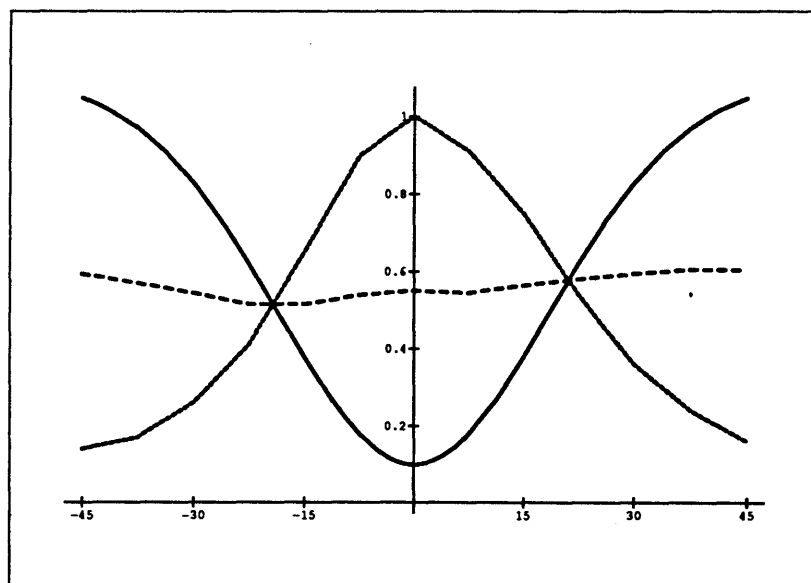

Figure 3.4: Radiated intensity of coated diffusion screen with collimated beam input from 300 mm in front center. Measurements were obtained of points spanning 45 degrees and -45 degrees from front center. A spot meter was used to take the readings. Also plotted is the inverse, offset Gaussian curve of the filter used to compensate for the radiated curve. The dotted line in the center represents the average values of the two curves added at each point.

efficiency of the H1 (due to substantial object self-interference) can be compensated for in the transfer step, a process recently discussed by Amitai and Friesem[1]. Experimental results from two-step 1-D and 2-D PDD HOEs gave rise to intensity distributions plotted in comparison to
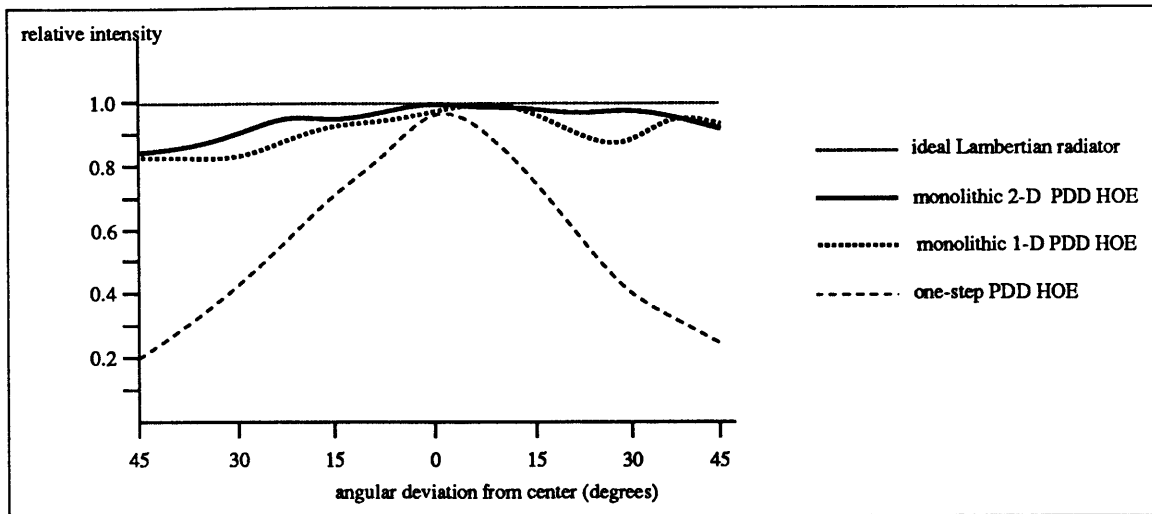
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Figure 3.5: HOE intensity falloff measured from front center.

ideal Lambertian and one-step results in Figure 3.5. Unfortunately, due to imperfections in the lenticular screen used for these experiments, and in the projection system used for masking the intensities, the noise level in the direction orthogonal to the direction of diffusion was higher than acceptable. Nonetheless, the two-step method promises to be a very effective means of stereogram producing PDD HOEs for specific, fairly constrained configurations.

Limitations

Although man of the properties that had previously made monolithic preferred-direction-diffusion HOEs less than ideal have been improved upon, there still remain several severe limitations to this approach. First, if the HOE is to be used in a stereogram object beam system without a collimator/louver film zero order blocking sandwich, the f/# of the system determines the reference beam and thus the illumination beam angles. This would not be a problem were we not projecting images onto the HOE that have limited resolution. Projection onto the inclined plane in an f/1.0 system with a 30 cm focal length is diagrammed in Figure 3.6.

HOE
focus

1.41 * x

x

film projector

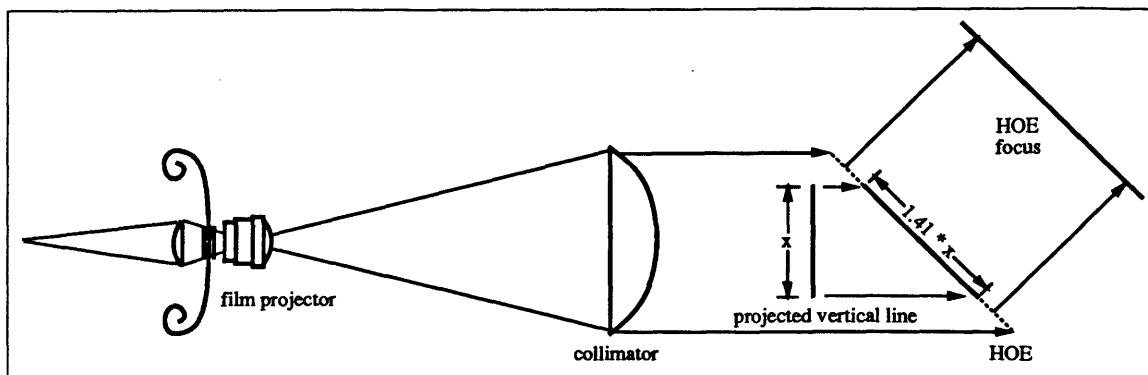projected vertical line

collimator

HOE

Figure 3.6: Stretching distortion due to projection on an inclined plane.

In this situation, the image must be shrunk vertically by a factor of 0.71 so that its projection onto the inclined HOE plane still results in a square image. The plane itself is inclined at 45 degrees so that the zero order of the illumination beam clears the projected slit image. If we desire an f/0.5 HOE with the same characteristics to be used in the system in a similar manner, the illumination angle increases to nearly 65 degrees and the resulting shrink factor increases to 0.5. This implies that the resolution of a square image would have to be doubled in one direction in order to retain square pixel projections onto the HOE plane. This is clearly an unrealistic expectation. Thus, unless an off-axis collimator and louver film sandwich is used to optically apply such a shrink factor without loss of resolution, higher speed monolithic HOEs are out of the question. This is not to say that such a system is not attainable. However, due to limitations in transmittance angles of available louver film, plus the need to make multiple optical elements, such a system does not lend itself to scalability or ease in manufacture.

Other limitations of the monolithic HOE approach also arise when scalability is considered. The two-step method implies the need for a diffusive element for recording the H1 that is at least as large as the planned stereogram projection screen size. This is relatively easy to acquire for isotropic diffusers, which are routinely made in dimensions exceeding one meter-squared. One dimensional diffusers such as lenticular screens, however, are more

difficult if not impossible to find in large sizes, thus rendering the two-step approach rather problematic for scaling. A one-step scale up is realizable with less difficulty, though it by no means promises to be an elegant solution. Finally, the premise of exposing meter-by-meter size monolithic HOEs in a very limited amount of space and with a modest amount of laser power at our disposal connotes a very arduous task and thus motivates research into more practical means of producing PDD HOEs.

Integral HOEs

The need for a scalable, low f-number system stimulates investigation into alternate methods HOE recording. Long, optically clean cylindrical lenses or more exotic beam-diverging optics would tend to be very expensive, difficult to produce, and unwieldy to manipulate. In addition, the need for exceptionally large collimating optics and plateholders, as well as for a large amount of lab space and a powerful laser, makes monolithic HOEs seem a risky venture at best. It is not clear that f-numbers lower than 1 can actually be achieved in the monolithic system without requiring impossibly large illumination angles and/or beam splitters for recording. The idea of creating an HOE by segmenting its aperture during recording emerged from this analysis as a potential solution.

Exposure Contiguity

It is not surprising that the premise of making a stereogram, or "integral" HOE would emerge from an image display stereogram tradition. Indeed, the relationship between the integral HOE technique and the monolithic approach is directly analogous to the comparison of a hologram and a holographic stereogram. It can be shown that just as a monolithic hologram diffracts wavefronts as if they were emanating from a physical object, a stereogram of the same object diffracts a piecewise approximation to the same wavefront. It is even more intuitive to show

that a continuous focus of, say, a long cylindrical lens can be composed of a number of discrete foci of much smaller cylindrical lenses exactly abutting one another. We exploit this simple hypothesis in the fabrication of integral holographic optical elements.

In general, an integral HOE is formed by making successive adjacent strip exposures of the field radiated by an optical element. The exposures are spatially sequenced, and in the simplest case each exposure is identical to the rest, allowing for a step-and-repeat exposure scheme that simplifies the system and reduces the amount of space needed to make the HOE. Due mainly to this segmented nature of the integral HOE, issues of exposure abutment, f-number and focal length, zero order suppression and focus characteristics can now be easily studied to determine the best format HOE for recording one-step stereograms. Of these four subjects, the first three concern the physical recording and subsequent performance of integral HOEs specifically, while the fourth begins to investigate characteristics of HOE focus and their effects on the stereograms that they will aid in creating.

Perhaps the most obvious concern when recording a segmented optical element is that of forming as close to a continuous element with a continuous focus as possible. Indeed, the exposure abutment issue has been an important one for display stereograms for quite some time due to its tendency to profoundly affect image fidelity. The most straightforward method for recording integral HOEs involves using a physical slit to delimit the separate integral exposures, in much the same way as a DeBitetto-type stereogram exposure is formed. In the case of an integral HOE, we replace the diffusion screen with a diverging optical element whose radiative field will subsequently interfere with a spatially filtered, collimated reference beam in each slit exposure. The holographic plate is translated a distance corresponding to the width of the slit between exposures. Contiguity in both the integral exposures and in the focus formed by conjugate illumination after processing is determined by how well the slit exposures abut one another and the relative width of the beam at the plane of the diverging element. Two important factors influence the contiguity of integral foci and exposures: one, consistent and perfectly matched slit width and translation distance, and two, the intensity distribution of light across

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the slit width. The first point is strictly governed by accurate machining of the slit, and reliable, repeatable translation. The second factor, however, implies that a very critical adjustment to the object beam is necessary in order to preserve perfect intensity continuity.

Figure 3.7 illustrates some potential geometrical relationships between the integral exposures and the foci they produce. The first example shows an idealized exposure with a collimated constant intensity distribution across the slit. If such a situation were realized, a continuous focus would be produced when the HOE was illuminated with its phase conjugate. Unfortunately, due to slit imperfections, diffraction at the slit aperture, and translation resolution limits, an exposure situation like that diagrammed in the first example is very difficult to achieve. The second and third examples show exaggerated versions of common exposure abutting errors, both resulting in discontinuities in the projected focus. Finally, the fourth example shows the most common scenario: substitution of a Gaussian intensity profile for the constant profile across the slit width. In this case a focus with a cyclical intensity profile is produced.

In order to better illustrate the physical slit mask integral HOE system, we will detail an example setup used for creating this type of HOE for the white-light alcove printer detailed by Krantz[19]. The system, documented in Figure 3.8 utilizes a 3 millimeter wide slit aperture placed 300 millimeters from a cylindrical lens with an aperture of 6 mm and a focal length of 6 mm. The intent is to create an f/1 cylindrical lens integral HOE with a focal length of 300 mm and an aperture 300 mm in width, so that 100 exposures are required. For this particular arrangement, a 45 degree collimated, spatially filtered reference beam was used. The plate was translated using a standard stepper-motor-slider which was controllable via computer interface. The integral HOE produced using this printer, hereafter termed 45 DEGREE CYL LENS diffracts real images of the focus of the object beam cylindrical lens side-by-side in space 300 mm from the HOE plane. This system produces an integral version of Huff's monolithic cylindrical lens HOE used for multiplex printing.
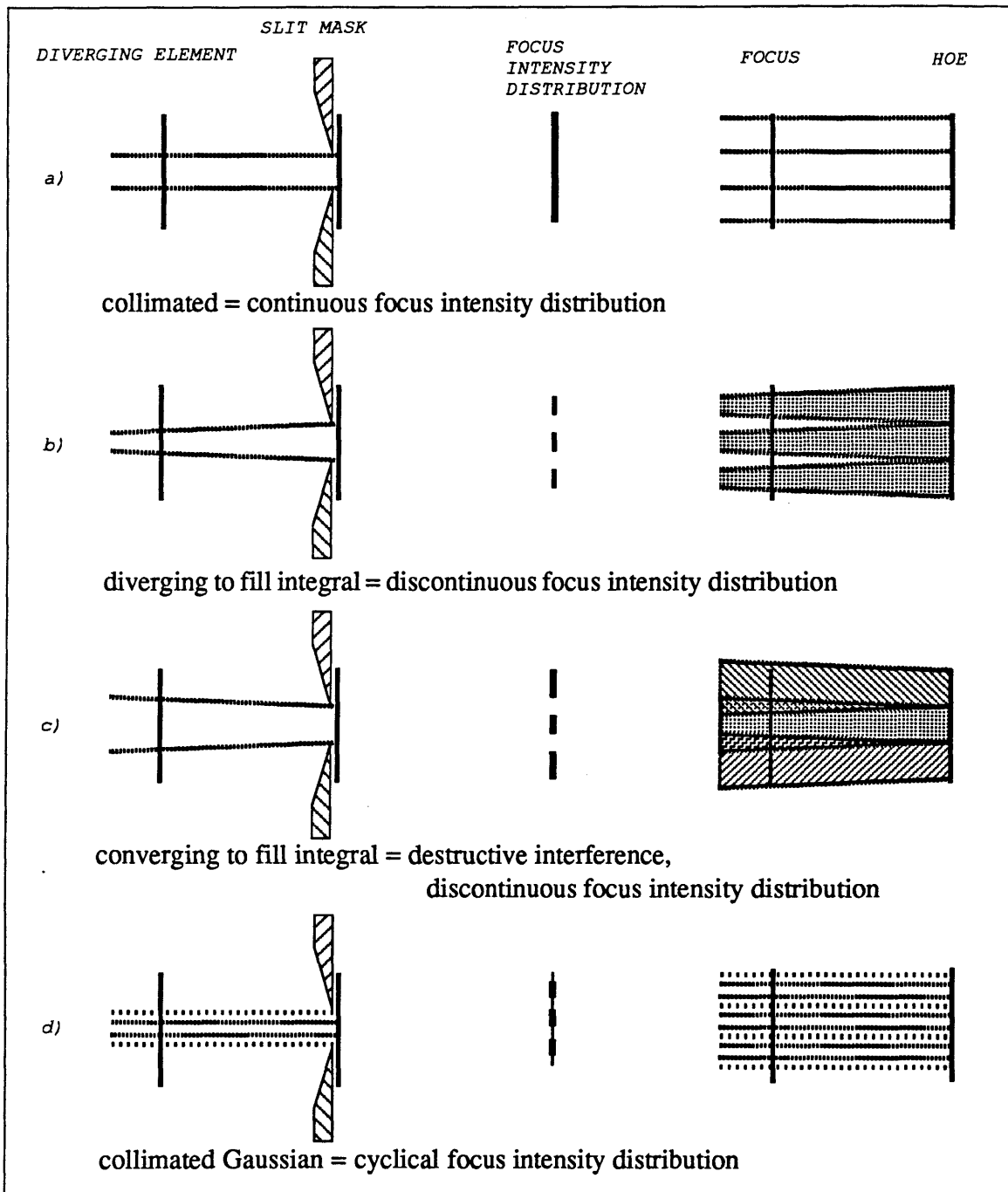
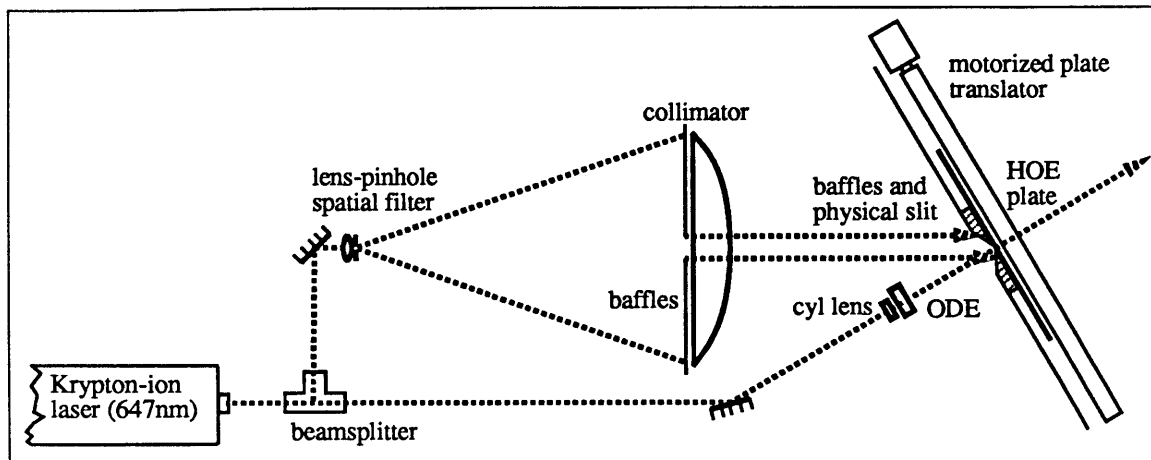Figure 3.7: Integral exposure/playback relationships

Figure 3.8: Setup for recording constant strip intensity distribution integral HOEs

The biggest concern in making the integral cylindrical lens is the continuity of the focus. If we ignore slit-edge diffraction effects, the focus of the HOE should theoretically be continuous when the the entire aperture of the HOE is illuminated with the phase conjugate (in this case collimated) beam if the following conditions are met: a) the intensity across the slit in each integral exposure is constant, b) the diverging optic is masked to exactly the same width as the slit, c) the intensity across that mask is constant, and d) the holographic plate is translated exactly one slit width per exposure. Due to the Gaussian intensity distribution nature of a laser beam, it becomes necessary to expand and re-collimate the object beam to insure condition (c) and by extension condition (d) or a pattern like that documented in Figure 3.7d results. If, for instance, the diverging optic aperture width is increased to compensate for falloff, a phenomena depicted in figure Figure 3.7c is observed in which diffracted images from neighboring fields destructively interfere with one another in the area of overlap, causing a discontinuous focus to be projected. The difficulty in producing a continuous focus using the physical slit mask stimulates research into alternate ways of abutting exposures and diffracted foci.

If we analyze the inherent intensity distribution across an undiverged laser beam, we can see that there may be promise of more accurately abutting integral exposures. The intensity

distribution can be modeled as Gaussian (in this case only the single dimension in which the integral exposures are abutted), and plotted, as in Figure 3.9a. A silver-halide based
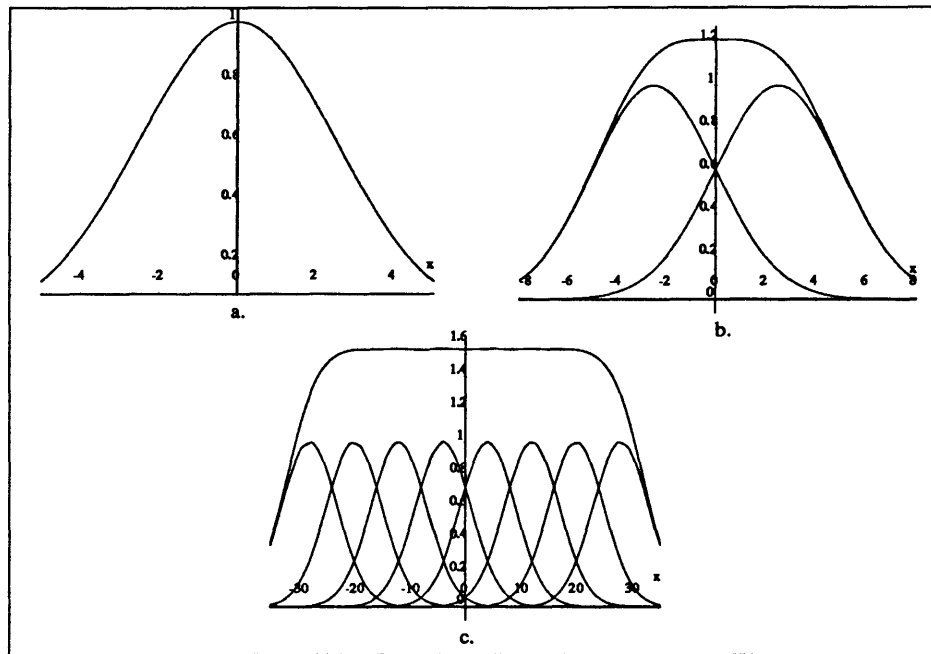
Figure 3.9: Gaussian intensity distributions and their summations.

photographic film exposed in the linear region of its H&D curve will record and preserve this intensity curve as a nearly identical density curve. Provided that both exposures are well under the saturation level of the holographic emulsion, two exposures with the same Gaussian falloff can theoretically be incoherently exposed slightly overlapping one another with a density superimposition given by the plot shown in Figure 3.9b. This presumes that the intensities of the two exposures in the areas of overlap are well under the material saturation level, so as to prevent application of non-linear effects including the well known "1/N rule" where N equals the number of temporally incoherent exposures. Thus, it is evident that by properly overlapping and abutting Gaussian intensity distributions we can produce an integral HOE with a nearly-constant density pattern, thus retaining continuity in exposure. A similar rule must be applied to the intensity distribution emanating from the diverging element in order to insure
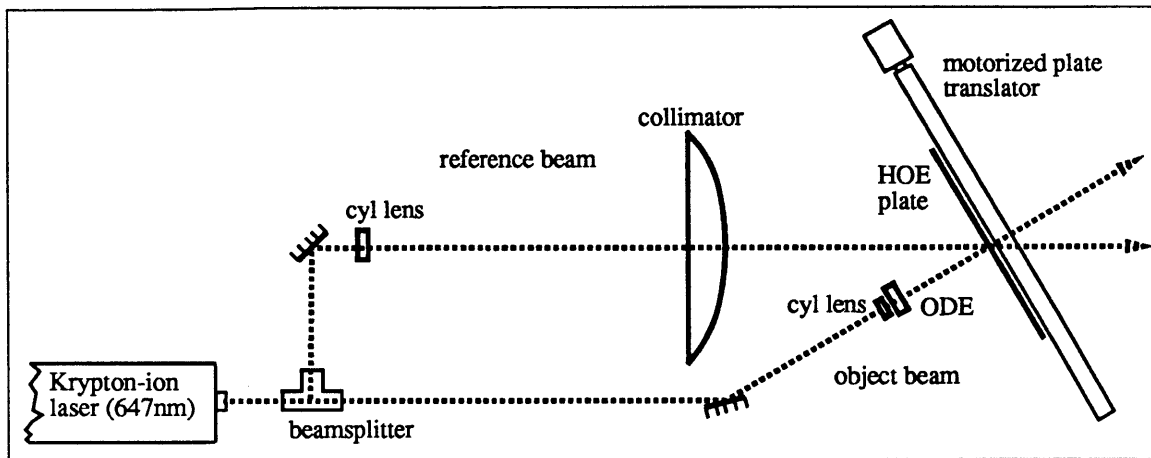
Figure 3.10: Setup for recording Gaussian strip intensity distribution integral HOEs

similar abutting of consecutive foci and thus create as continuous a focus as possible. The resulting focus ideally would have an intensity distribution illustrated in Figure 3.9c.

In addition to the continuity factor, the Gaussian overlap integral HOE is much easier to produce due to its lack of a fixed-width slit mask. In an example setup, diagrammed in Figure 3.10, the spatial filter in the reference beam has been replaced by a small cylindrical lens which diverges the light vertically, but not horizontally, and most importantly preserves the Gaussian intensity distribution in the beam's cross-section. This collimated "fan" beam is then interfered with the object beam which emerges from an unmasked beam-diverging element. Provided the pathlengths of the two beams are approximately the same, and no divergence in the cross-wise dimension has occurred, the beams overlap each other almost perfectly. A holographic plate placed at the region of overlap will record the Gaussian cross-slit distribution, and, with proper translation between successive exposures, a nearly continuously-exposed HOE results.

The main benefits of the Gaussian HOE lie in the fact that it is much easier to produce than the slit mask type, due to the non-existence of the slit and thus the freedom to translate the plate whatever the required distance between exposures. In addition, the obvious desire

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for a continuous focus motivates use of this method, which far exceeds the capabilities of the slit-mask counterpart in producing just that. The Gaussian approach is not without challenging problems however, the foremost of which is the fact that the continuity of the exposure density is directly dependent on exposure energy. This factor can be experimentally determined rather easily, however, with a simple test wedge in which a given exposure energy is kept constant while translation distance is varied. Another difficulty is the fact that this type of HOE exposure is much more sensitive to scatter and stray light due to the fact that no physical slit exists to filter it out. This implies that the diverging element must be fairly noise free, while still retaining its efficiency. This is not a problem when a cylindrical lens is used, but a one-dimensional diffuser potentially poses problems. Despite these difficulties, the benefits seem to outweigh the disadvantages, enabling very continuous foci to be produced using this method as compared with the slit mask method in Figure 3.8.

F-number and Zero order Suppression Issues

Just as in the case of the monolithic HOE, the effective f-number of the integral HOE is determined by the f-number of the optics used to make it. For the monolithic HOE, however, we are faced with the problem of acquiring a 30 centimeter-long low-f-number cylindrical lens, while for the integral counterpart, only a 5 millimeter-long optic is necessary due to the piecewise accumulation of exposures. Needless to say, it is much easier to produce an optically clean 5mm-apertured low-f-number optical system than a 30 centimeter one. An additional difficulty with low-f-number diffractive optical systems in general is the need to record with an extremely high reference beam angle in order to insure that the object beam optics do not obstruct the reference beam path to the hologram plate. If we wish to use a reference beam angle no greater than, say, 45 degrees, the lowest f-number achievable without reference-object obstruction in a monolithic HOE is 1.0. In an integral HOE, however, a much lower angle can be used for virtually any f/number desired since the aperture is built of narrow strip exposures. Figure 3.11 demonstrates that the geometry of the system no longer depends on fitting a long

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Figure 3.11: a. $f/1.0$ monolithic HOE system showing f-number limitations due to angular cutoff of the reference beam by the object beam. b. $f/ \lesssim 1.0$ integral HOE system with much larger f-number range available due to slit-wise exposure
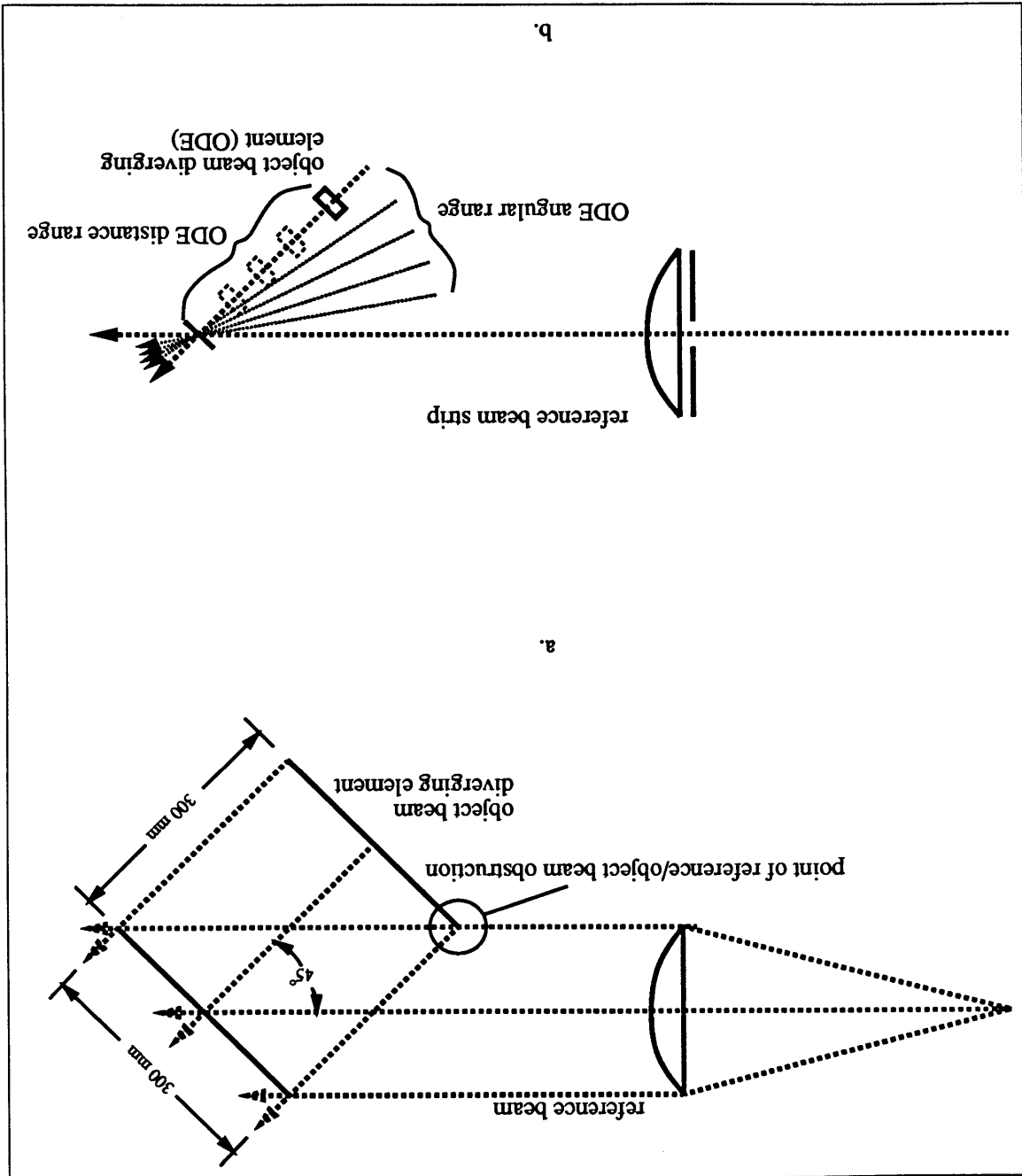
optic in the object beam, thus blocking the reference beam. Instead, both beams are in the form of narrow vertical strips, greatly increasing the possibilities for a faster HOE optical system. For one-step reflection Ultragrams, we wish to duplicate the results of a two-step system, which, as illustrated before, has an effective f-number of 0.5. We can choose nearly any angle we wish for an integral HOE reference beam, whereas a 65 degree angle would be necessary for the monolithic case. The integral HOE arrangement is consequently much more accommodating to a wider range of geometries governing the optical element constraints.

Given that an integral HOE system allows us the possibility of making a low f-number, low-illumination angle optical system, it becomes necessary to investigate the characteristics of such a system from the point of view of HOE recording. A later section will be devoted to characteristics of low f-number systems from a image playback perspective.

Although the integral system diagrammed in Figure 3.11b is very flexible insofar as illumination angles and object beam diverging element (ODE) distances are concerned, there still remains a problem of zero order diffracted beam suppression. A diagram of an $f/0.5$ low-illumination angle (in this case 25 degrees) integral HOE is shown in Figure 3.12 to help demonstrate this impediment. zero order suppression is not an issue in the monolithic case, since the illumination beam does not obstruct the diffracted beam by virtue of the geometry used in recording. Thus, the reference beam angle constraints of the monolithic system automatically insure that illumination/focus superposition does not occur. In the integral HOE case, however, where the same constraints are not adapted, an illumination/focus superposition can occur, thus complicating the problem of producing a low-illumination angle, low-f-number HOE.

As discussed previously in context of limitations of the monolithic HOE system, a low illumination angle HOE is desirable due to issues of projected image resolution. Because of these constraints, the lower the achievable illumination angle, the less the predistortion necessary to project the stereogram component images. The complication arises due to the fact that the smaller this angle, the larger the area of illumination/focus superposition becomes.
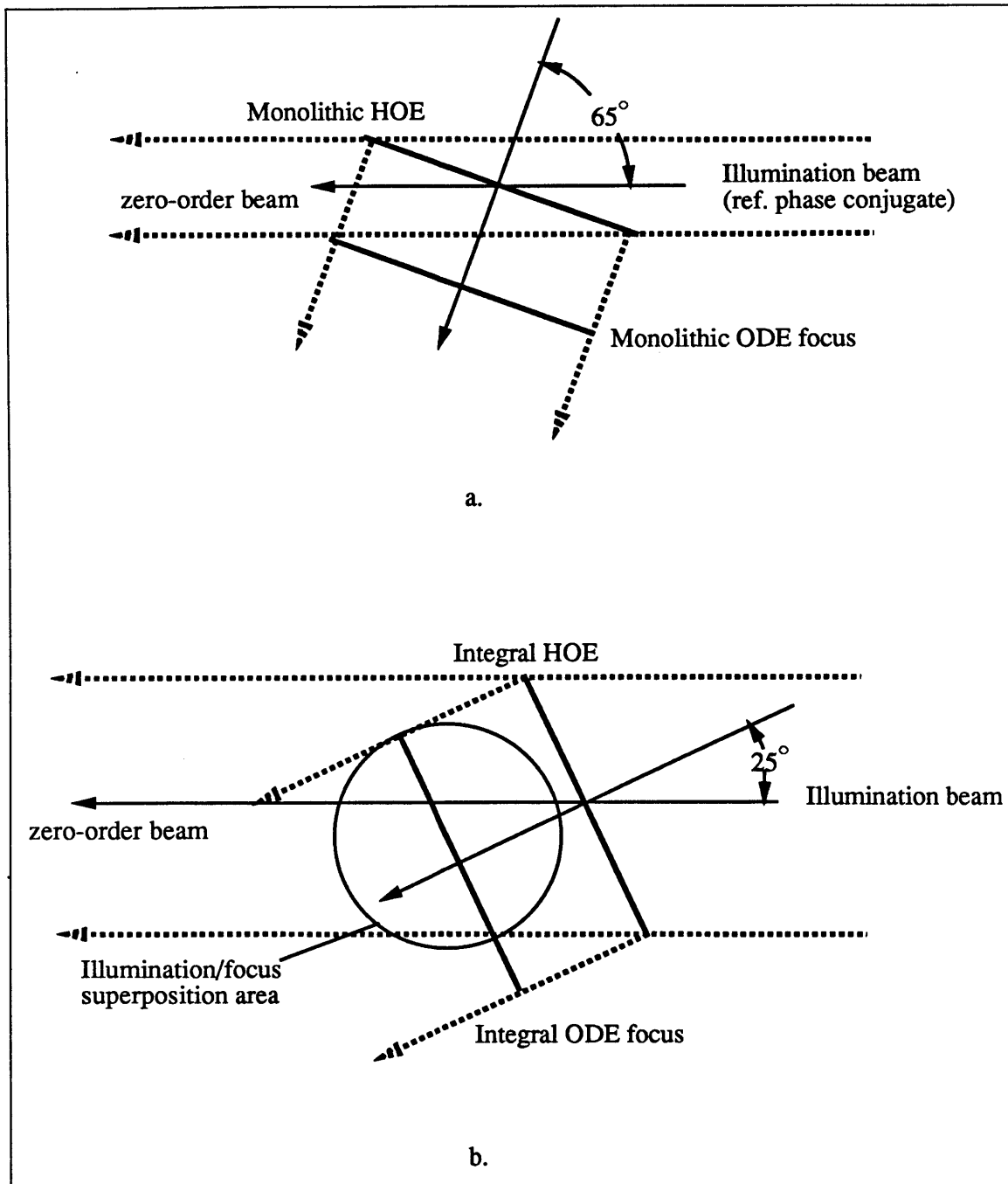
Figure 3.12: a. $f/0.5$ monolithic HOE displaying large angle needed for zero order clearance
b. $f/0.5$ integral HOE showing area of illumination beam/focus superposition

Thus it becomes necessary to somehow suppress the zero order diffracted beam to prevent it from overlapping the HOE focus.

There are a number of possible solutions to the illumination/focus superposition dilemma. If the HOE could be recorded on an extremely efficient material, such that nearly 100% diffraction efficiency could be achieved, this beam superposition becomes a non-issue due to the lack of a zero order beam. Unfortunately, it is difficult to achieve exactly 100% diffraction efficiency even in materials like dichromated gelatin, not to mention the silver halide material used in these experiments. The matter is complicated by the fact that even if we were able to achieve such efficiencies, the signal to noise ratio would most likely drop, a characteristic that is not beneficial for re-imaging HOEs like this one. Interestingly enough, however, if the material were sufficiently able to diffract a significant portion of the light into the first order beam, a completely in-line element might possibly be produced, thus eliminating the need for off-axis reconstruction and the associated difficulties. Experiments were made with an such in-line approach, but low diffraction efficiencies coupled with optical distortions fated this as a futile cause. Still, if a material were someday produced that could be capable of such diffraction efficiencies with low noise, the in-line system would be the most ideal.

Another, much more realizable method of eliminating illumination/focus superposition is to somehow physically block the zero order beam from intersecting the focus. The material needed for such a task would have be transmissive to light at impinging at zero degrees, and opaque to light impinging at some angle on either side of zero degrees. The material would be placed between the integral HOE and its focus parallel and close to the HOE. The zero order light passing straight through the HOE would then be intercepted by the material, but the light diffracted by the HOE would be allowed to pass through. Fortunately, a material fitting this description is commercially available as of the time of this writing. "Light Control Film," more commonly referred to as "louver film" because of louver blind-like structures within the plastic

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layer, is made by 3M company in a variety of different transmissive angles[1]. Since the smallest louver film transmissive angle currently available is 26 degrees, we have chosen 30 degrees as the reference/object beam angle for recording our integral HOEs. This angle is sufficient to preserve the resolution of our component images with a small amount of predistortion required due to the angular projection. A cross-sectional diagram of the louver film used is shown in Figure 3.13.
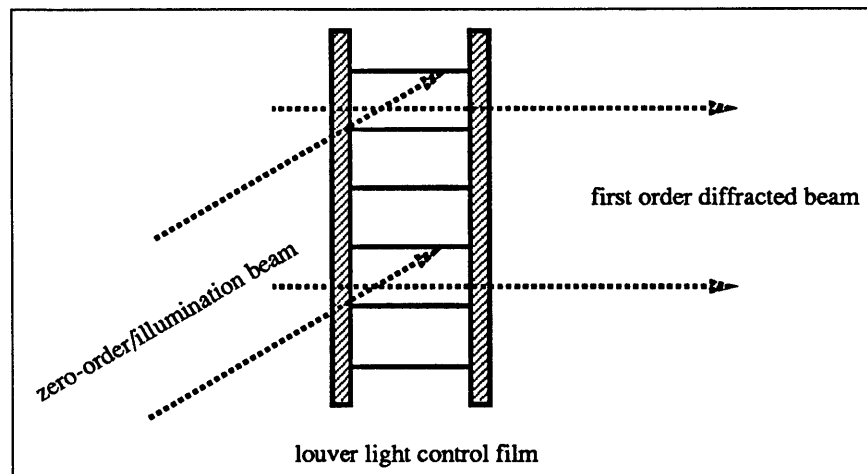

Figure 3.13: Cross-sectional diagram of louver light control film showing zero order suppression properties.

Louver film methods for zero order suppression have been used before in slightly different approaches. Benton uses a louver film layer in his composite collimator/preferred-direction diffuser HOE[4]. This system requires that a second HOE is made to collimate the incoming diverging light, and direct it into the PDD HOE through the light control film at the louver angle, and finally to the focus of the system. The louver film in that case suppresses the zero order beam of the collimator, not the PDD HOE element itself. An almost identical system was used to produce the Alcove stereogram[2]. In another, somewhat related application, DeBitetto uses louver film to block the zero order white illumination light in a dispersion compensation

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[1]Available from 3M Optical Systems Division, Bld. 225-4N-14, St. Paul, MN 55144

scheme[9]. The louver film approach is favorable due to this film's general availability, and, in the system described here, its inclusion in a system with only one optical element. This simplicity will allow for a fairly straightforward scaleup of the integral PDD HOE system to meter-by-meter without the need to make multiple optical elements.

Figure 3.14 details the reconstruction of the integral foci using the louver screen. The
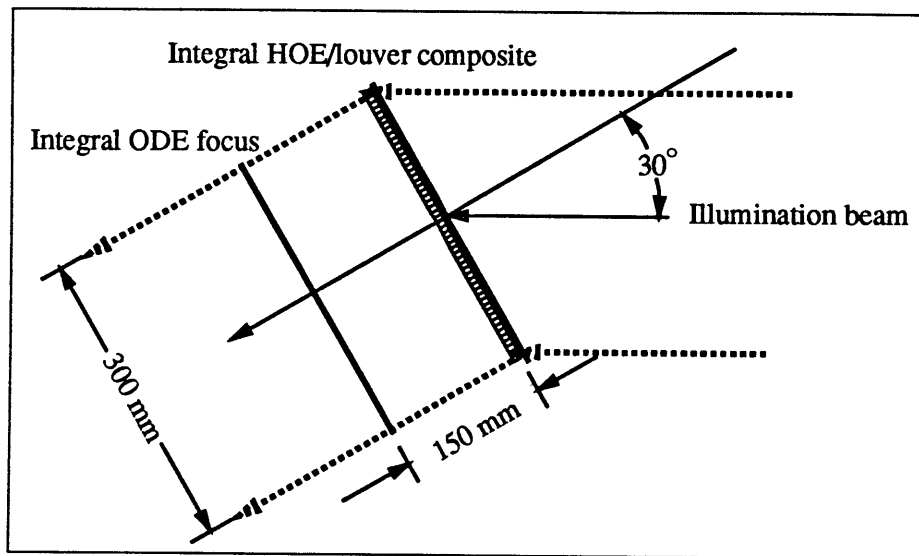
Figure 3.14: Focus reconstruction using louver light control film to suppress zero order beam.

screen is laminated directly to the HOE emulsion in order to keep the system simple and reduce the amount of space necessary to construct the printer.

HOE Focus Characteristics

The final integral HOE issue to be discussed is the specific type of ODEs used and the quality of the HOE focus for our particular purposes. Due to the "picket fence" effect experienced in previous experiments, one of the goals of this research was to find a way to eliminate slit structure in one-step stereograms. In order to eliminate it, one must first study it to determine

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its origin.

The problem stems from the fact that one-step image-plane stereograms are not made up of angularly multiplexed images *per se*. A standard white-light viewable reflection stereogram, or even a two-step Ultragram for that matter, has the plane of angularly multiplexed images on the image-plane, with the slits to which each of the overlapping images are mapped floating above or below the image plane. In a one-step system, the roles are reversed; the slits in the white light viewable image rest on the image-plane, while the spatially multiplexed perspective images lie separated from the plane by some distance. If we consider the fact that at the image-plane, the highest resolution available horizontally is the resolution of the slits, we can see that an image-plane display is not really the best arrangement for a one-step image. In fact, since the spatially multiplexed images rest on the HOE plane (which is analogous to the ground glass plane in the two step case), the plane of highest resolution (the horizontal resolution of the graphics) is actually the HOE plane. Nevertheless, the one-step system, if it is used as an image-plane stereogram, retains this image-plane resolution loss as an intrinsic property. Since this research centers around recording the stereogram with a diverging reference beam point source, then reconstructing with approximately the same point source (direct reference illumination), the HOE image lies behind the image-plane of the final stereogram, thus complicating our efforts to solve the resolution problem.

An additional factor to consider when placing the integrals on the image-plane is the fact that source size will no longer blur neighboring exposures together, as was the case in a DeBitetto transfer. This factor makes the decision as to what type of focus should image the object beam onto the stereogram integral plane that much more critical.

In previous research in this realm, a cylindrical lens type of ODE was used to create the integral 45 DEGREE CYL LENS HOE, thus synthesizing a piecewise large aperture small f/number cylindrical lens in the HOE itself. This type of HOE, when illuminated with its phase conjugate, produces a very small focus, on the order of the diffraction limit, assuming perfect

replay of the small ODE cylindrical lens focus used to record it. The cylindrical lens is a convenient model with which to record side-by-side perspective images: it has power only in one direction, it has no vertical diffusion associated with it, it is relatively easy to produce, and it is very efficient with small amounts of light. Cross used these characteristics of large cylindrical lenses to develop the Multiplex (tm) stereogram, a one-step transmission stereogram made up of 1080 perspective views spaced 3 per degree around a cylindrical piece of film[7]. Regardless of how convenient the cylindrical lens is, from the perspective of each integral exposure, analyzing specifically the effect on the image-plane, we find that a diffraction limited focus fills only a very small part of a 1 mm wide integral slit exposure. This fact is illustrated in Figure 3.15, a photograph of integral exposures in a 1mm slit spacing produced by the 45
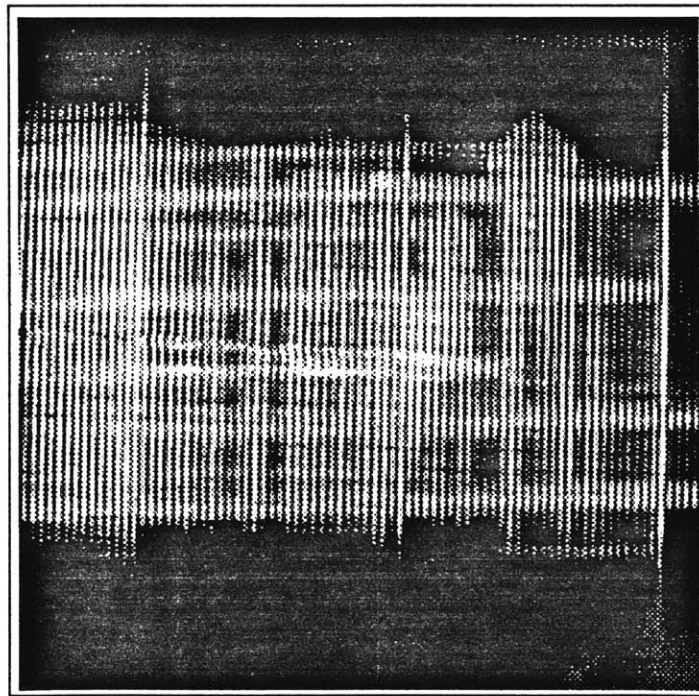

Figure 3.15: Integral exposures formed by 45 DEGREE CYL LENS HOE in a one-step Ultragram test pattern. Note the "picket fence" effect.

DEGREE CYL LENS HOE. In a case such as this one, the focus occupies an estimated 1/20th

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of the width of an entire integral. Even if the focus is moved so that the it does not rest on the plane of the emulsion during exposure, the focus is still imaged in space, and will always appear in an integral exposure as the brightest part of the integral, always occupying an area significantly smaller than the width of the integral. This seems to be the major source of the "picket fence" effect visible in one-step stereograms, including the Multiplex (tm). It is more noticeable in flat-format images due to the tendency to use image-plane formats in which at least part of the image passes through the slit plane of the stereogram. Because the resolution at the image-plane is already substantially reduced by the nature of the recording, the "picket fence" effect further degrades that part of the image, disrupting the three-dimensional effect and distracting the viewer.

The are a number of possibilities available for improving the quality of the integral exposures to prevent "picket fencing." One is to simply increase the resolution of the printing to match, or more closely match the width of the cylindrical lens focus. A second solution involves replicating the diffraction limited cylindrical lens focus by either placing a low-frequency grating in the object beam system before the film, or by recording the HOE itself with a replicated cylindrical lens focus. A third option is to insert a one-dimensional diffuser behind the film, and a rectangular aperture in the projection lens with a width the same as that of an integral exposure. Finally, one could record a cylindrical lens analog of a one-dimensional diffuser with a mask as wide as the desired integral focus. This final solution would provide an aperiodic diffused focus in a system in which the ray bundle converging from the plane of the HOE would never become narrower than the width of the integral slit, and then diverge again. Because the main thrust of this thesis lies in investigation of HOEs for stereogram printers, the research documented here concentrates on the solutions which can be recorded into the HOEs, although the other passive solutions are acknowledged also.

Because we wish to produce an HOE focus of some finite width greater than that which a cylindrical lens produces, we must consider what it means to have an optical system with less than diffraction-limited performance in one direction. If we consider, for example, an HOE

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which would produce 20 identical foci in the width of the slit, each identical to and with the width of a cylindrical lens focus, we could do a much better job filling the integral exposure with information. Twenty identical foci spread over a 1mm can be interpreted as being formed from twenty cylindrical lenses superimposed over one another and translated 1/20 mm from the position of each other. Logically, if such a group was able to exist in space, and a collimated beam was input into it, the foci produced would be displaced from one another by 1/20 mm. Ideally, an HOE with these characteristics could be produced with a lenticular screen ODE with 20 lenticules/mm pitch. Material realities tell us that lenticular screens with very low effective f-numbers (high dispersion angles) are difficult to produce, and screens with pitches much higher than 6 lenticules/mm are not readily commercially available, but these are not the only limitation to this approach. Due to the fact that the lenticules are merely small cylindrical lenses with identical focal lengths, each producing a focused line source displaced a very small amount from its neighbors, the field radiated by a coherently-illuminated area of lenticules exhibits a low frequency interference pattern. This interference pattern, with a spatial frequency on the order of 10 cycles/mm, makes it difficult to produce a constant intensity diverged pattern in the integral exposure, thus recording the low frequency pattern on the entire aperture of the HOE. This is, of course, unacceptable. We need to either destroy the periodicity caused by neighboring lenticules, or randomize the relative phase of the light exiting each lenticule focus, while at the same time increasing the pitch of elements in our ideal ODE.

Because lenticular screens of the desired pitch are not commercially available, it becomes necessary to consider methods of producing a lenticular-like one dimensional diffuser that will disrupt the periodicity and phase of the diverged beams while simultaneously diffusing the light in one direction only. The solution to creating such diffusers was first proposed by Meyerhofer in 1973, and involved making an in-line hologram of a lenticular screen[20]. In such a system, diagrammed in Figure 3.16, each lenticule focus acts as a reference beam for all the others, allowing the simultaneous recording of a very large number of spatial frequencies. The interference patterns range from 10 cycles/mm to over 2000, depending on the parameters
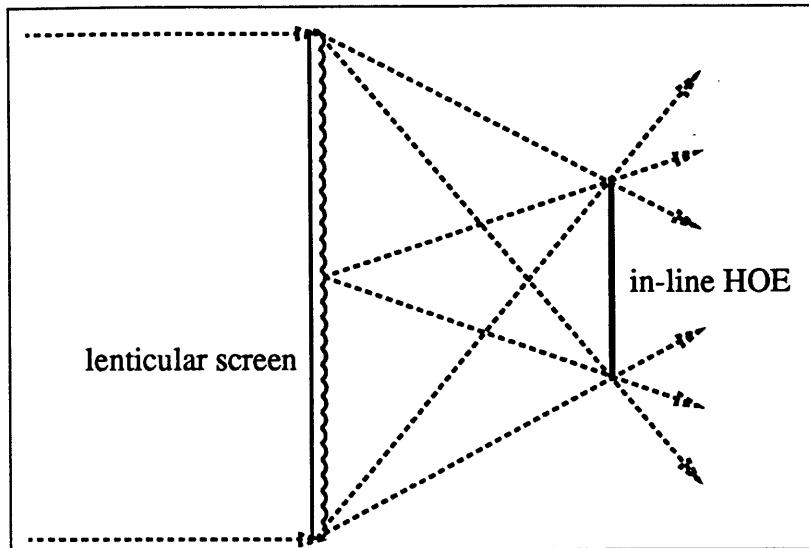
Figure 3.16: Setup to record in-line one-dimensional diffusers.

of the system, and they are all spatially multiplexed onto the plane of the recording hologram. A magnified view of the 1-D fabricated 1-D diffuser is shown in Figure 3.17, as compared to the isotropic coated screen diffuser at the same magnification. The diffusion properties of the bleached lenticular in-line hologram make it well-suited as an ODE for the integral HOE system. The multiple cylindrical lens model of analysis can still be applied to show that the lenticular in-line behaves as desired. If we were to illuminate the lenticular in-line with a collimated beam through a mask with a width equivalent to the diffraction limit, the field that would be projected would be equivalent to that of a cylindrical lens with an extremely short focal length. By widening the slit mask, we uncover more "micro lenticules" which also diverge the light in a similar manner. Because the aperiodicity of the light diverging elements and their sheer number per unit area in this 1-D diffuser is so much greater than in the case of a standard lenticular, the superposition of diverged fields gives a randomized speckle output, rather than a regular interval interference pattern. This is also probably due to the fact that the lenticular screen used to make the in-line is far from perfect, allowing for enough phase variations to destroy strict periodicity in the divergence pattern.
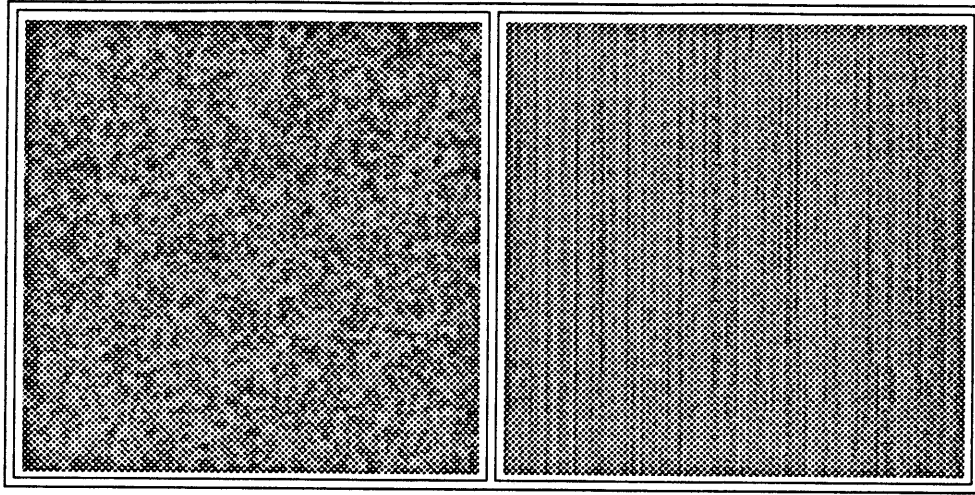
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Figure 3.17: Isotropic diffuser magnified by approximately 100X (left) compared with fabricated 1-D diffuser also magnified by approximately 100X (right).

We can use the general 1-D diffuser HOE as an object beam diverging element for creating the integral HOE. By illuminating the 1-D diffuser with an undiverged beam, it is possible to achieve divergence in the object beam which is equivalent to a f/0.5 cylindrical lens. We can then mask the input beam with a slit that is the width of the integral exposure we wish to achieve with the integral HOE. We can additionally create a Gaussian focus, so that the same theory used to create the HOE can be used to create the stereogram with the HOE. By inputting a wide beam that converges to the integral exposure, we can actually produce a quasi-isotropic preferred-directional diffuser. These different options are diagrammed in Figure 3.18. Four preferred direction diffusers were created exploiting these various characteristics: a 1mm physical slit focus HOE with a constant intensity across the slit image, a 1/2 mm Gaussian focus, a 1 mm Gaussian focus and a 2-D preferred-directional diffuser (PDD) for use in a two-step system. The varying Gaussian foci were produced by diverging the beam with a cylindrical lens just prior to its passage through the 1-D diffuser. By moving the lens closer or further away from the diffuser, a the "spot-width" which became the focus of the integral HOE could be changed.

The resulting integral HOEs have the property that they act like superimposed large-
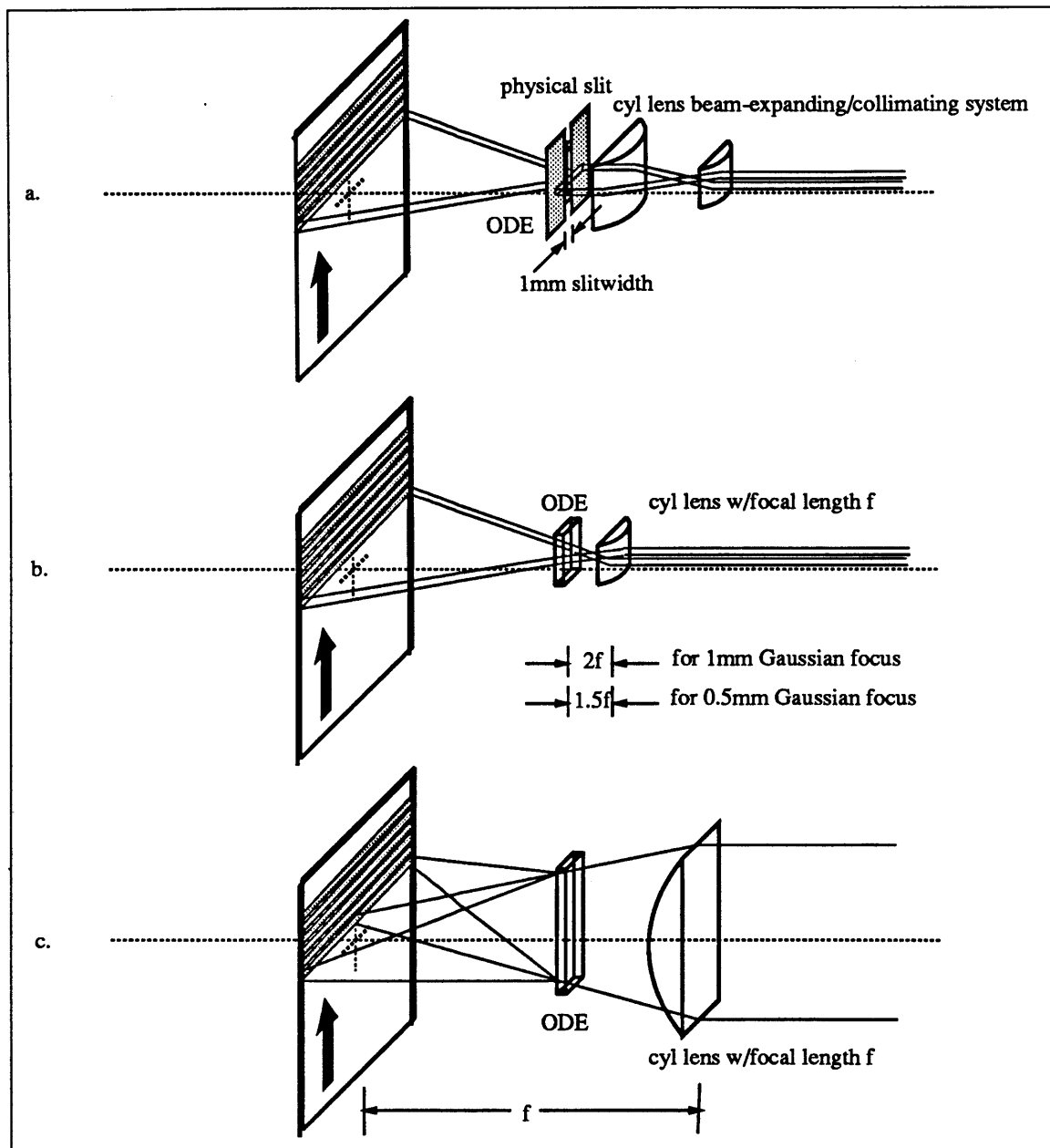
Figure 3.18: Integral HOE recording for a. 1 mm-wide constant intensity focus, b. Gaussian foci (focus widths noted), and c. 2-D diffuser

aperture cylindrical lenses, but they also can be modeled as preferred-direction diffusion screens that only diffuse light perpendicular to the direction of the focus slit. Thus, we have created 1-D PDD HOEs (one-dimensional preferred-direction diffusion holographic optical elements) with optical characteristics similar to the monolithic 1-D PDD HOEs, but in a more scalable format and with a much lower illumination angle. These HOEs exploit the issues of ease of manufacture, scaleup potential and cosmetic and optical correction to produce the foci for integral HOEs.

Chapter 4

Stereogram Printing Issues

One-step image-plane stereogram techniques have been under development for approximately 20 years, starting first with Cross' Multiplex(tm) system and progressing through the Alcove stereogram to current variations on these two themes. The issues involved in recording one-step images condense to a series of compromises of format, view-angle, chromatic dispersion and image-plane resolution. This chapter will discuss one set of parameters centered around the 1-D PDD HOEs described in the previous chapter. This is by no means the only method available to produce one-step images, nor is it necessarily the best, but it fits the general requirements for an effective image dictated by the success of two-step Ultragrams documented in Chapter 2.

The optical setup used for testing the HOEs is diagrammed in Figure 4.1. For these tests, it was decided to use a one-step volume-reflection type exposure in order to best mimic the two-step Ultragram case in a single step. The optical system in the reference beam was variable in order to manipulate the intensity distribution across that beam and thus test that variable's effects on stereogram exposure. The measurements of the printer were based on an exact 1/3 scale-down of the meter-squared one-step system referred to in Chapter 2 so that the results could be interpolated up easily to that printer's size. The image used for each HOE test was a
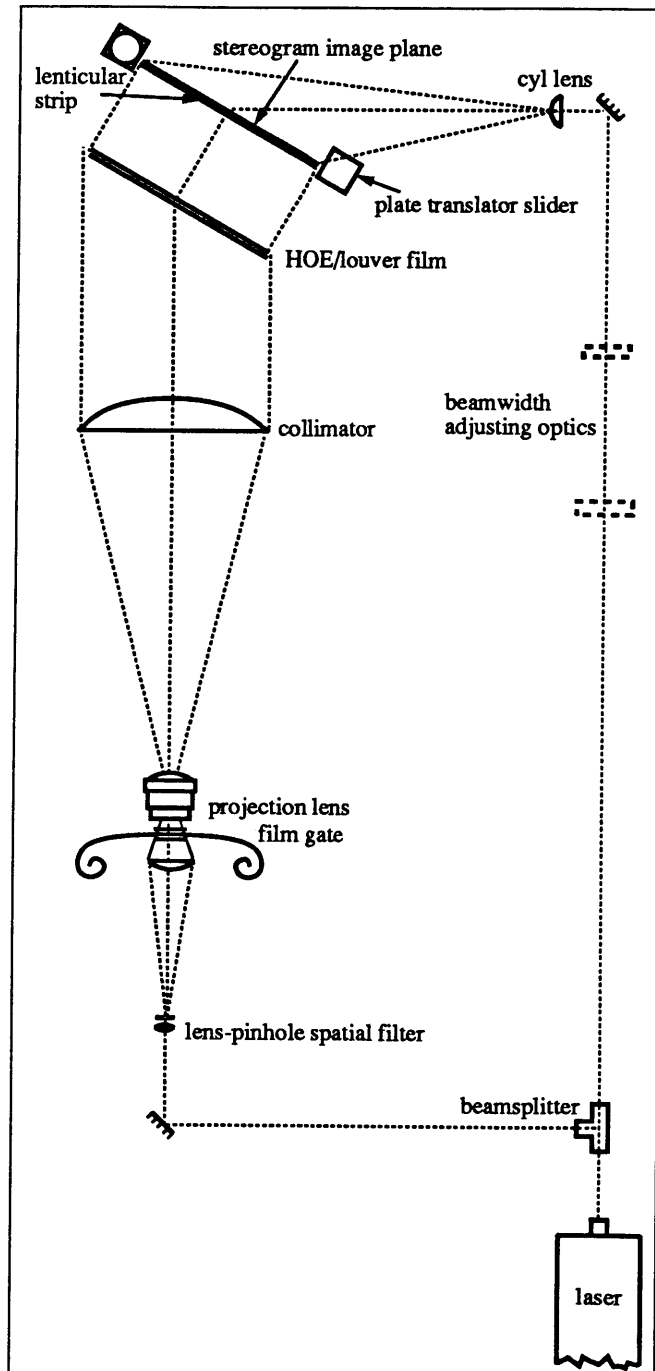
Figure 4.1: One-step Ultragram setup for testing f/0.5 holographic optical elements.

test pattern depicting six equal-sized grids made of anti-aliased lines and oriented in space as documented before and after predistortion in Figure 4.2.
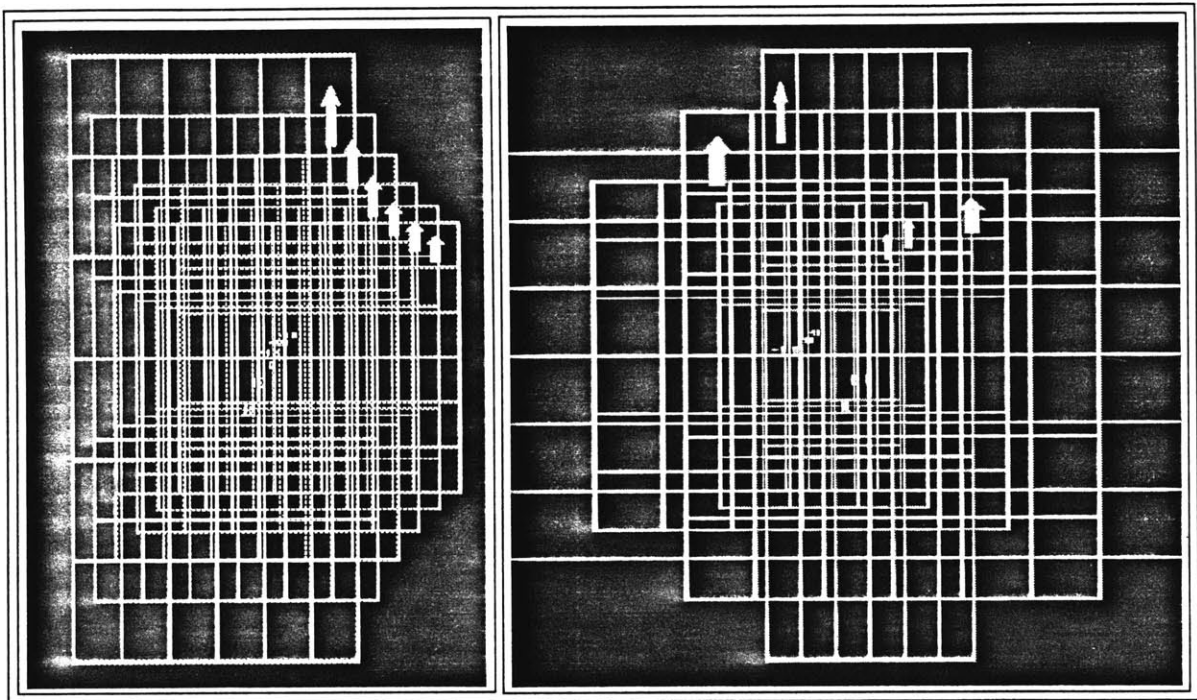
Figure 4.2: Image used as test pattern for HOE stereogram printer. L–before predistortion, R–after predistortion).

The perspective images were generated and predistorted as detailed in Chapter 2. A total of 300 predistorted views were generated for the first test set, and thus the final image size was designed at 300 mm x 300 mm with a total designed depth of 750 mm. A second test set with 600 views was also generated specifically for the 1/2 mm-wide focus HOE, preserving the 300 x 300 aspect ratio.

Exposure Contiguity

The issue of exposure contiguity is even more important in recording the stereogram image than it was in recording the integral HOEs. As discussed earlier, the one-step stereogram suffers from the fact that the plane of angular multiplexing is not on the image-plane, thus limiting the image-plane resolution to that of the integral exposure resolution. It is important, therefore, to take into consideration the psychophysical effects of the exposure columns on the image-plane, just as it is in the case of a low-resolution CRT display, for example. These effects are more pronounced in the case of a three-dimensional image because of their tendency to distract the viewer and reduce the depth-illusion by attracting one's gaze to the image-plane itself. Displays such as the Multiplex(tm) and the Alcove do not suffer as much from this effect because their image volumes are usually centered around a point that is far from the hologram plane. The Ultragram, by virtue of its image-plane-bisecting object placement, must be as free from these artifacts as possible. It therefore becomes necessary to reduce the so-called "picket fence" effect caused by underfilled integral exposures and then evaluate the trade-offs associated with that reduction.

The ideal image-plane one-step Ultragram would be composed of integral exposures formed by a diffraction-limited converging optic, such as a large diameter, short focal length cylindrical lens. Unfortunately, such a system would require a very high density of integral exposures, implying a very large number of component images and thus a significantly long image generation and stereogram printing time. Since one of the primary motivations of recording stereograms is to reduce quantities of needed information, this approach would constitute a step in the wrong direction. For our experiments, we have somewhat arbitrarily chosen an integral exposure spacing of 1 mm, and in some cases 1/2 mm, because this implies a reasonable number of images that have to be generated and printed.

As discussed before, the one-dimensional diffusing method is not the only way to fill the integral spacing with object beam information in the stereogram recording step. Two so-
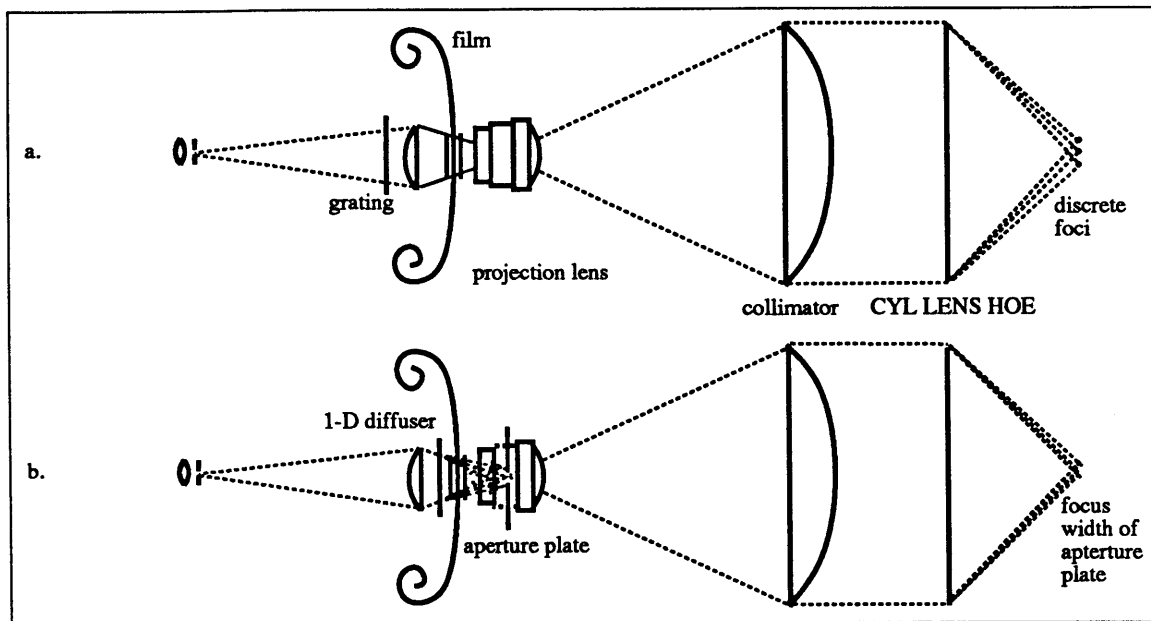
Figure 4.3: Passive focus expansion techniques

called "passive" integral-filling techniques are diagrammed in Figure 4.3. In Figure 4.3a, a cylindrical lens HOE is used to converge the object beam light in conjunction with a grating "upstream" which serves to multiply the focus and spatially separate each duplicate focus. Such an approach would require a grating of very low spatial frequency, due to the very small angle of foci divergence desired, and the sheer number of replicated foci needed to ideally fill the 1 mm integral width. Possibly a better method, diagrammed in Figure 4.3b, is to insert a non-periodic one-dimensional diffuser in the object beam just prior to the point where the beam intersects the image on the film. A rectangular aperture with a 1mm width is then placed into the projection lens which in turn images the aperture onto the stereogram plane. Because the diffusion angle is so great, and the diffuser is non-periodic, a continuous focus is spread across the 1mm wide integral, completely filling it. Both of these methods are effective, however they are somewhat wasteful of light and, in the case of the diffuser-aperture method, require some machining of the lens and aperture in order to be actualized.

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By designing and optically encoding similar diffusing effects in a cylindrical-lens like HOE, we have produced an object beam system that has the same integral-filling properties, but is more conservative with the limited amount of light available. A microscopic view of the intensity components which make up the focus of the 1-D PDD HOEs is shown in Figure 4.4. These photographs show that the foci are composed of segmented micro foci, which, when
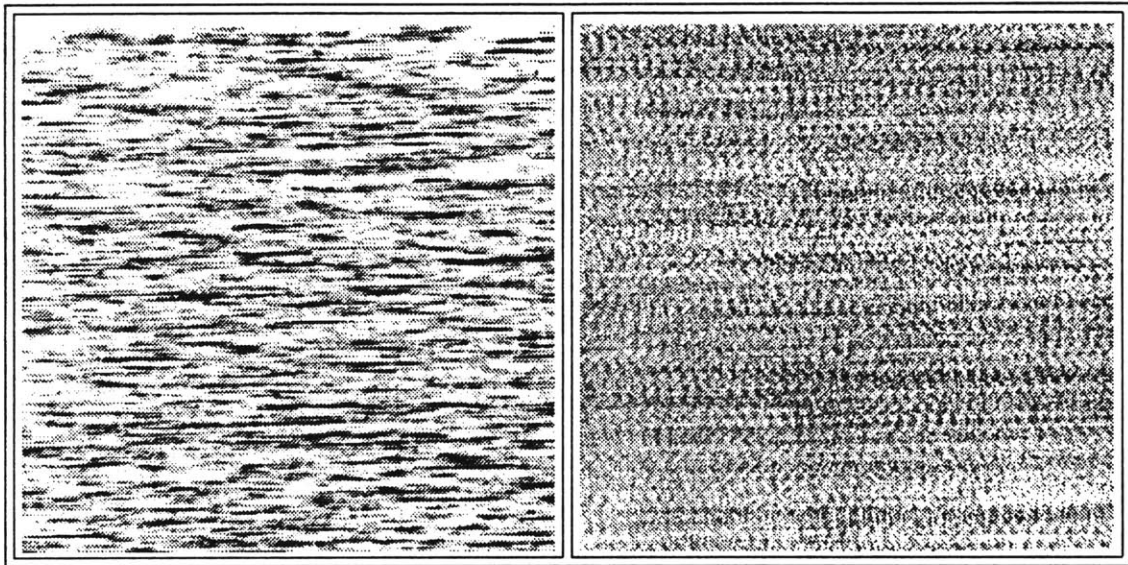
Figure 4.4: Microscopic view of GAUSS HOE focus (left) and RECT FOCUS HOE focus (right) on holographic plate. Focus lines run horizontal in both cases.

bleached will have significant optical power in the direction orthogonal to the focus line. Three of the integral HOEs detailed in Chapter 3 were designed to fill the integral exposure width, while at the same time experiment with the problem of then abutting neighboring stereogram exposures to further reduce the "picket fence" artifacts.

The first HOE was recorded with a expanded beam across a 1mm wide slit placed at the plane of the 1-D diffuser. This HOE, hereafter referred to as RECT FOCUS HOE due to the resemblance of its focus intensity distribution to a rectangular mathematical function, is most analogous to the diffuser/aperture approach documented above, since the focus produced has

Figure 4.5: Reference versus object beam intensity profiles for two RECT FOCUS HOE experimental exposures

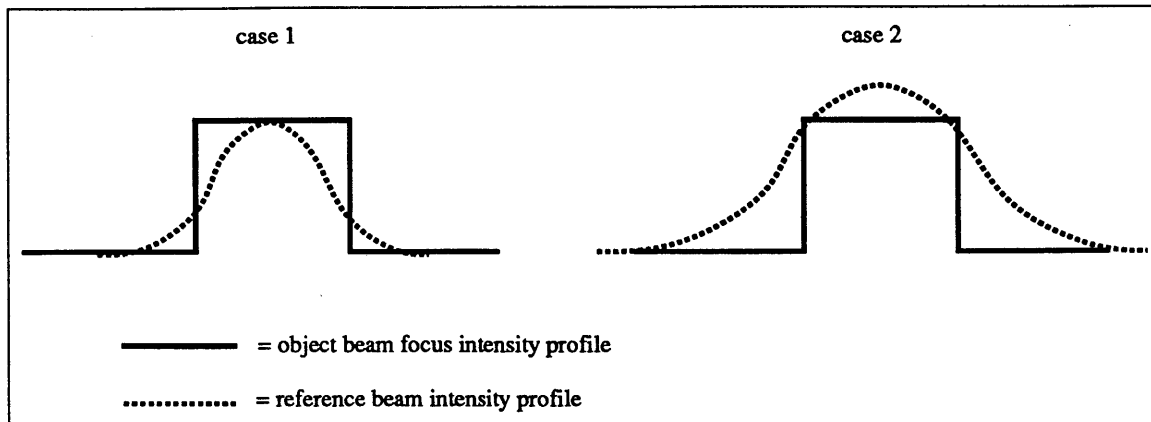hard edges and aperiodic constant intensity diffusion across its width. The existence of hard edges, however, incites the need for a very accurate abutting of consecutive exposures, and a matching of the reference beam intensity distribution to the rectangular intensity function. The first challenge is a mechanical one and, as such is relatively simple to overcome. The second, however, is much more difficult, due to the Gaussian nature of the reference beam in the slitwidth dimension. Other than using a physical slit which rides directly on the emulsion to minimize diffraction effects, a rectangle function intensity distribution collimated reference beam seems impossible to produce.

Experiments on the RECT FOCUS HOE were made using Gaussian intensity distributed reference beams of two different widths as diagrammed in Figure 4.5, to determine if the fidelity of the final stereogram image is degraded by the intensity mismatch. The first case, in which nearly all of the Gaussian intensity curve fits into the integral width, produced underfilled integral exposures, causing very evident picket-fence artifacts. In the second case, when the Gaussian was expanded such that neighboring reference beam exposures overlapped as in diagrammed (c) in Figure 3.9, reference beam and diffracted field contiguity was significantly improved.
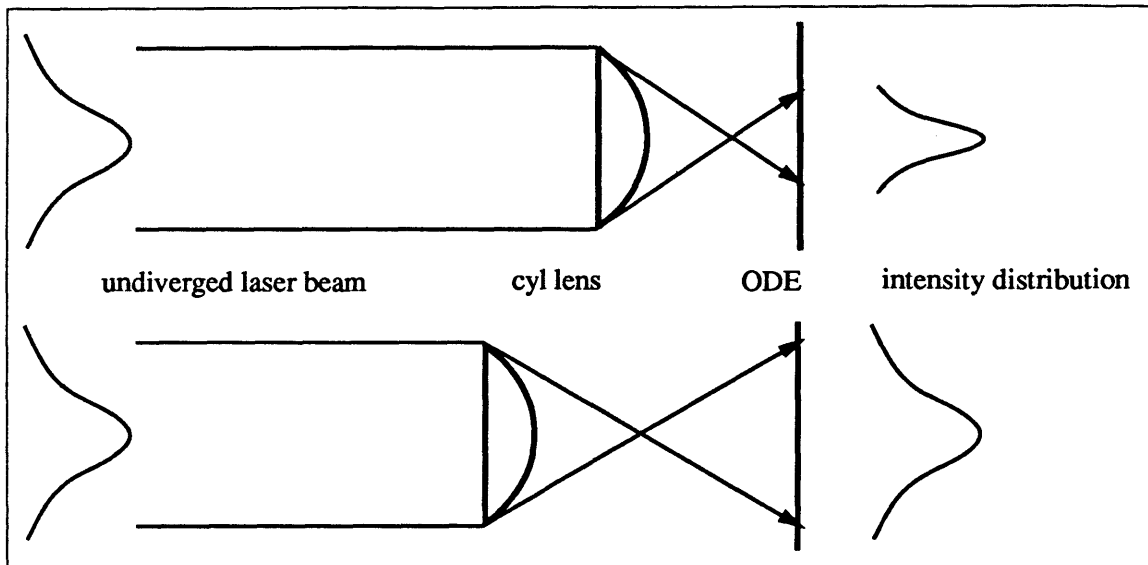
Figure 4.6: Cylindrical lens used to change width of Gaussian HOE focus

The two other HOEs tested were recorded with 1 mm and 1/2 mm wide Gaussian foci, and are referred to as 1MM GAUSS HOE and 0.5MM GAUSS HOE respectively. The width of the Gaussian was approximated and assumed to be measured at the $1/e$ point on the intensity curve. Due to a lack of instrumentation allowing the measurement of the precise width of the curve at this point, we relied on previous experiments to approximate it. These experiments indicate that the Krypton laser beam measured at $1/e *$ maximum intensity at a distance of approximately 3 meters from the aperture of the laser has a width of approximately 1 mm. By inserting a cylindrical lens in the object beam as documented in Figure 4.6, the effective Gaussian "width" could be changed. The Gaussian focus is much more adaptable to the stereogram printing system, because the reference beam in that system has a similar Gaussian falloff property. Thus, a simple adjustment of the anamorphic optics in the reference beam allowed perfect matching of the Gaussian intensity distribution of the object and reference beams.

Exposing the test pattern with the 1MM GAUSS HOE gave excellent exposure continuity results, documented in the photograph in Figure 4.7 as compared to those of the RECT FOCUS
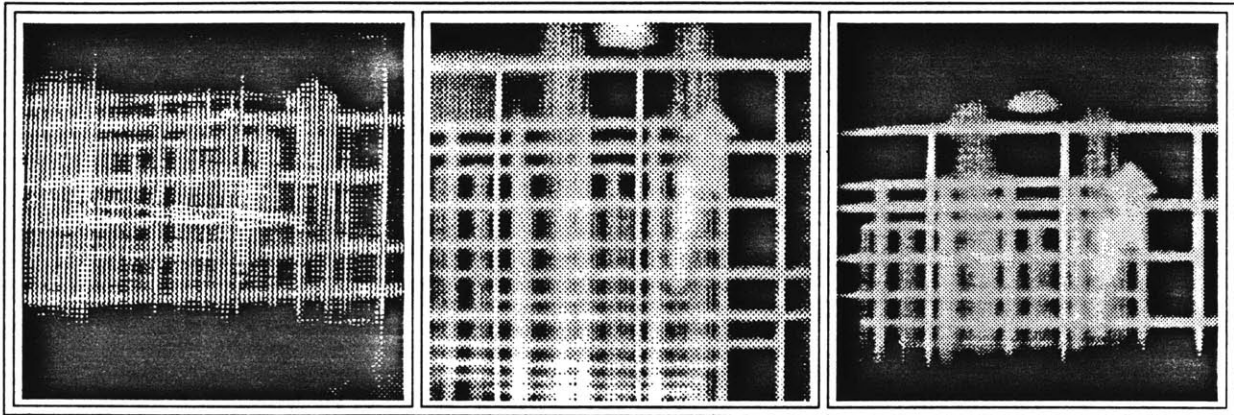
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Figure 4.7: Close-up views of resulting Ultragram one-step images made using (l-r) 45 DEGREE CYL LENS, RECT FOCUS, and GAUSS FOCUS HOEs.

HOE and the 45 DEGREE CYL LENS HOE. It is very interesting to note that the continuity does not seem to be affected to a great extent by the substitution of the Gaussian distribution for the rectangle distribution. This is probably due to the fact that the information within the focus represents a massive replication of the information at the HOE plane, as if many phase-randomized cylindrical lenses projected it simultaneously. Thus, overlapping consecutive object beam exposures as in the case of the 1MM GAUSS HOE and abutting consecutive object beam exposures as in the RECT FOCUS HOE case produces the same contribution across the expanse of the stereogram, intensity-wise, and consequently also diffraction-wise. The Gaussian method still seems favorable due to the ability to easily match the reference beam, and also in view of the difficulty in producing rect slit foci of varying widths.

A final test was produced using the 0.5MM GAUSS HOE, in order to display the ease of scaling the focus width down, and to better approximate the wavefronts of the imaged object by increasing the sampling frequency. This test was also the first image to include bandlimiting, (to be discussed in detail in the next section) in which vertical lines expand in width with distance from the HOE plane. The effects of bandlimiting and increased integral plane resolution made a significant difference in the quality of the image and the much-improved exposure contiguity. In

addition, since the focus was much smaller in the 0.5 MM GAUSS HOE case, the exposure time decreased substantially, allowing for a faster run time for the stereogram shot. The bandlimited half-millimeter test can be compared to the 1 mm non-bandlimited test in Figure 4.8 It appears that the ideal case, then, is bandlimiting with high integral resolution.
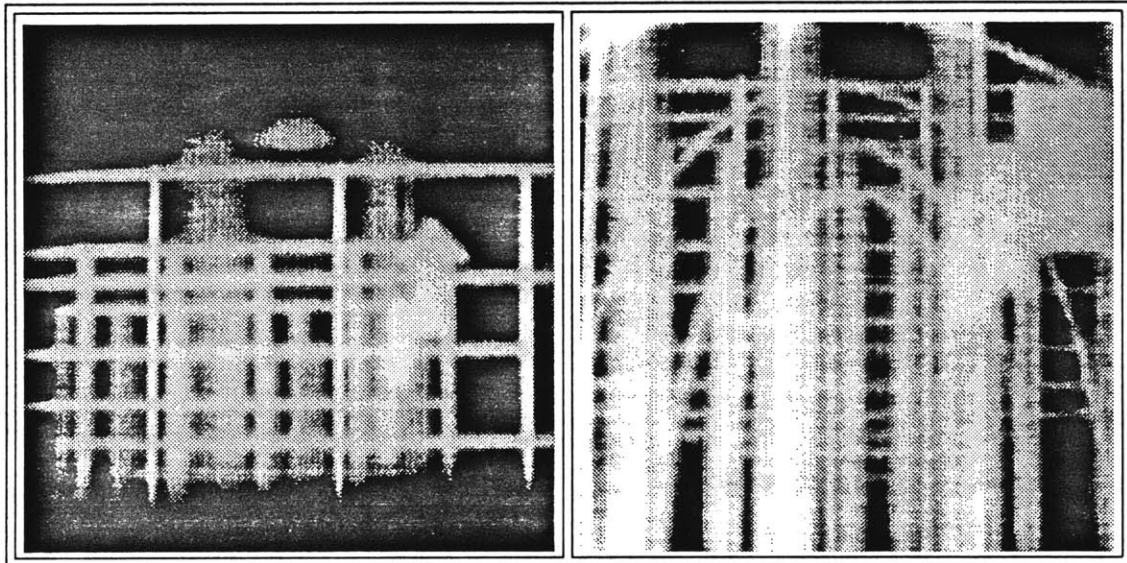
Figure 4.8: Close-up views of integral exposures of 1 MM GAUSS FOCUS HOE Ultragram and 0.5 MM GAUSS FOCUS HOE Ultragram.

Bandlimiting Considerations

Bandlimiting is a relatively new subject in the realm of holographic stereograms, having only recently been adapted into the subject from the computer graphics and signal processing fields by Halle[13]. A good deal of the material in this section therefore is expansion and application of Halle's work with emphasis on the role of the imaging in reducing and eliminating inter-view aliasing.

Stereograms, like any other discrete-sampling technique, are subject to aliasing artifacts
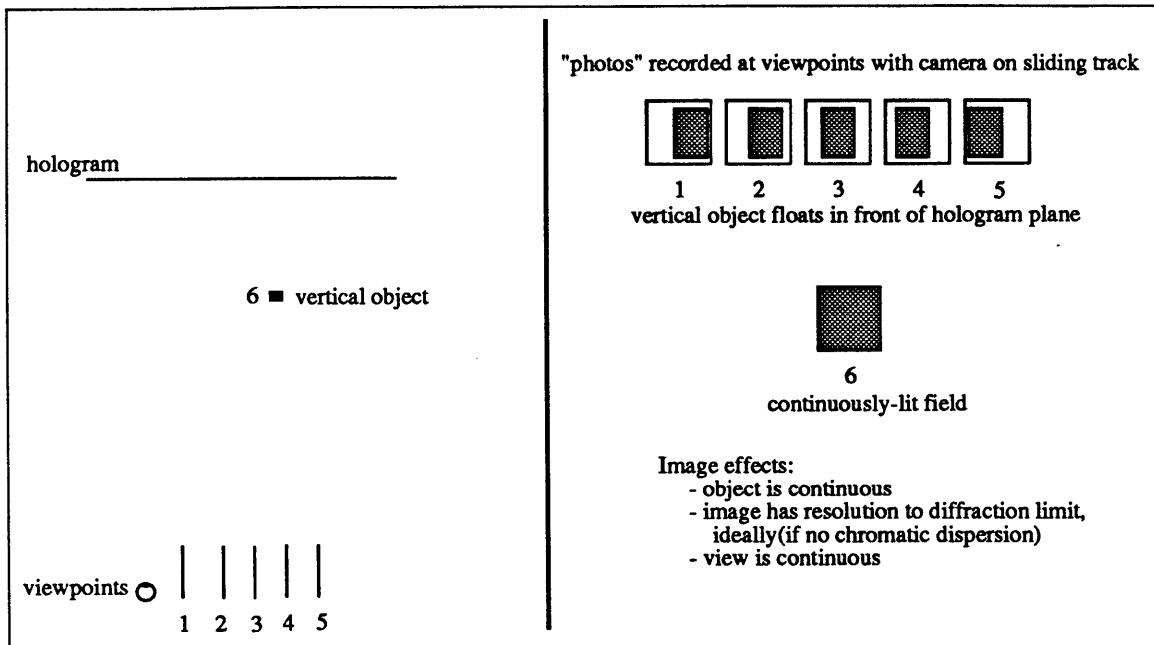
Figure 4.9: Comparison of point imaging in a hologram and a stereogram showing aliasing artifacts

due to the finite width of sample elements and a lack of continuous sample placement positions. If we compare a hologram of an object (Figure 4.9) to a multiplex-type stereogram of the same object (Figure 4.10), the aliasing effects of sampling become apparent. In this case, as the viewer's eye moves across the plate a distance equivalent to one slit-width, the image point in the hologram moves continuously, while the same point represented in the segmented stereogram master can only be in one of two positions, corresponding to the two sampled views represented in that segment. The effect on points close to the stereogram image projection plane is small, due to the lack of significant motion parallax for those points as recorded from the intended viewzone. However, points far from the projection screen suffer significantly, since their parallax perspectives virtually "hop" from one position to the next corresponding to the two sample views recorded, rather than flowing continuously, as in the hologram case. The result is a discontinuous reproduction of the imaged point, disturbing the depth effect of

Non-bandlimited, cylindrical lens HOE

HOE   a   b / image elements

integral exposure plane

6 vertical object

viewpoints ○
1 2 3 4 5

"photos" recorded at viewpoints with camera on sliding track

1 2 3 4 5
line source at infinity--tracks eye

6
image plane picket fence

Image effects:
- severe picket fence
- sharp image edges
- infinite depth of field
- inter-view aliasing
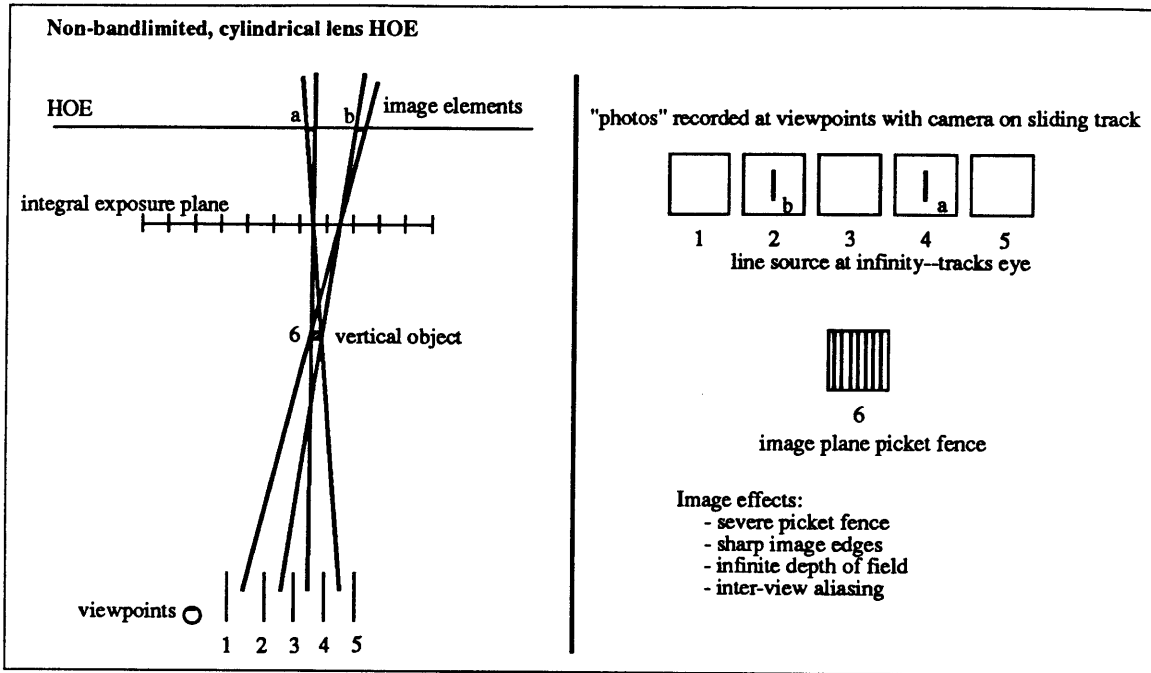
Figure 4.10: Representation of a vertical object, such as a column, by a one-step Ultragram recorded with a cylindrical lens object beam HOE. "Photos" of what is seen from the viewpoint positions 1-5 are diagrammed to right. Viewpoint 6 represents what the viewer sees when his/her eye is placed within the object image.

the image by virtue of the aliasing artifact.

One-step Ultragrams also suffer from lack of adequate slit-plane resolution to represent extremely deep objects. Just as in the standard stereogram case, the plane of least noticeable inter-view aliasing is the projection plane which is the plane of highest resolution and most angular multiplexing in the image volume. Standard recording of one-step imagery using a cylindrical lens or similar HOE, as diagrammed in Figure 4.10, causes extreme aliasing artifacts, due to the wide gaps between integral exposures, and the fact that individual points are perceived as collimated, originating from an infinitely far distance away, rather than from the projection screen. If the projected images were produced for diffraction-limited integral spacing and integral size and the cylindrical lens foci were properly abutted the aliasing artifacts would be

minimized. Unfortunately, such a arrangement would require rendering many more images (approximately 20 times the amount currently rendered), and thus defeat many of the benefits of stereogram sampling offers over pure holography. Another problem with the cylindrical lens integral exposure technique is the fact that it is impossible to image points which lie near the integral exposure plane, but in between exposed areas as shown in Figure 4.11. Thus,
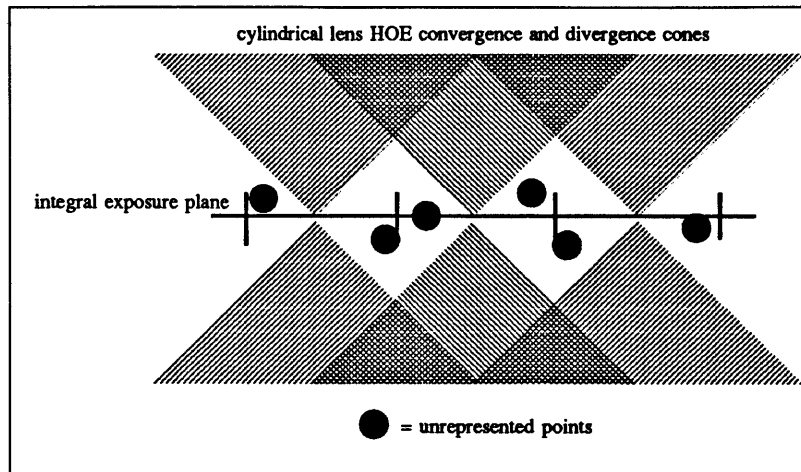
Figure 4.11: Points in the image volume not represented nor imaged by a cylindrical lens or cylindrical lens HOE in the object beam.

the non-bandlimited cylindrical lens HOE imaging system is a worse-case scenario of possible configurations.

A better projection system is represented by one using the 1-D preferred-direction diffusing HOE for object beam projection. In this case, diagrammed in Figure 4.12 information from the image elements fills the integral segment width. From viewpoints 1-5, virtual images of the image elements can be seen, as if they are positioned behind the integral exposure plane by the distance separating that plane from the HOE. As the viewer moves side to side, the object seems to be made of vertical bars, rather than being a solid field. This is the essence of the so-called "picket fence" artifact, resulting from the fact that although the integral abutment is correct in this case, the sampling limitation on the image volume depth gives rise to aliasing. In this case,

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Non-bandlimited, 1-D diffusing HOE

image elements

HOE

integral exposure plane

vertical object

6

viewpoints

1 2 3 4 5

"photos" recorded at viewpoints with camera on sliding track

1 2 3 4 5

discrete virtual images of image elements

6

improved picket fence

Image effects:
- vertical image edges softened
- reduced picket fence
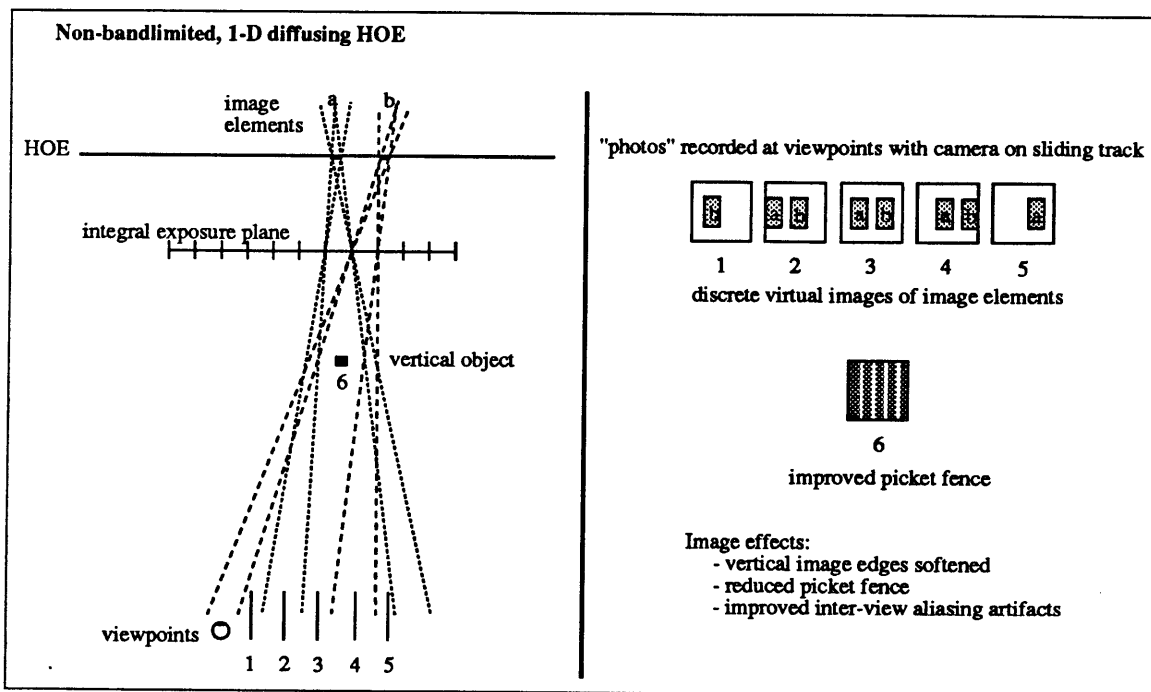- improved inter-view aliasing artifacts

Figure 4.12: Representation of a vertical object by a one-step Ultragram recorded with the 1-D preferred-direction diffusing object beam HOE.

Bandlimited images/1-D diffuser HOE

HOE

image elements\\ a \\/ b \//

"photos" recorded from viewpoints with camera on slider track

integral exposure plane

1   2   3   4   5

larger image elements
=> continuously visible virtual images

6 █ vertical object

6

picket fence eliminated

viewpoints ○
1 2 3 4 5

Image effects:
- loss of depth of field
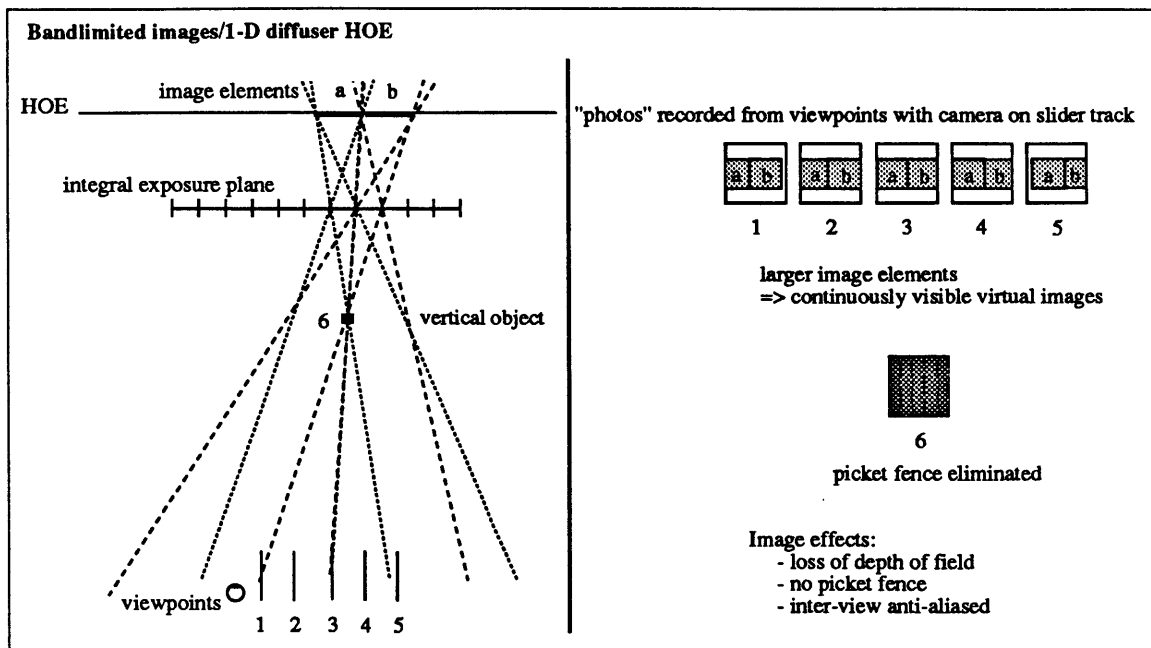- no picket fence
- inter-view anti-aliased

Figure 4.13: Representation of a vertical object by a one-step Ultragram recorded with the 1-D PDD HOE and bandlimited image elements

however, the aliasing cannot be termed simply as "inter-view", as Halle describes[13], since it is present in even single views of the image. Instead, we see an "inter-integral" aliasing, because in the case of an Ultragram the viewer looks through many slit exposures at once. View 6 in Figure 4.12 once again represents what the viewer sees when his/her eye is placed in the object image. The "picket fence" artifact is improved to the limits of the graphics, rather than to the limits of the optical system, as was the case in the cylindrical lens system. The ideal view 6 would be a continuous field, rather than a series of columns. Nevertheless, the optical system is better suited to the rendered images and the stereogram geometry than in previous cases.

There is a final improvement that can be made to insure the complete removal of inter-integral aliasing in the one-step Ultragram image. Figure 4.13 shows a case in which the image elements have been expanded in width to provide a bandlimited sampling for imaging onto

the integral exposure plane. The increased width allows for a continuous representation of the vertical object image, and thus eliminates picket fence artifacts cased by inter-integral aliasing. In a sense, the image volume becomes "anti-aliased" in a depth sense, allowing a viewer at the object position (view 6) to see a continuous field of light. The result of inter-integral anti-aliasing is analogous to depth of field effects seen in two-dimensional photography. Here, object points which are near the HOE plane remain sharp, possessing the resolution of the graphics at that plane, while object points behind and in front of that plane are blurred according to depth. It is important to point out that no information is lost in the anti-aliasing process, although removal of the picket fence artifact seems to have a psychophysical effect in perception vertical image element sharpness. Photographs of the cylindrical lens HOE case, the 1-D PDD HOE, and the bandlimited 1-D PDD HOE are shown in Figure 4.14.

Chromatic Dispersion

One of the most critical compromises confronted by printing one-step Ultragrams with an f/0.5 object beam imaging system is that of chromatic dispersion. In the context of one-step imaging we will treat horizontal and vertical dispersion individually, since their effects are noticeably different in the image.

In the case of the reflection Ultragram printer, vertical chromatic dispersion, with the potential of being far more severe than the horizontal due to the off-axis vertical angle of illumination in that direction, is controlled by placing the vertical diffusing plane at the stereogram image-plane. Placement of the vertical diffusing plane at the HOE focus (image) plane can be accomplished in three ways. In the case of a transmission stereogram recorded with a cylindrical lens or 1-D PDD HOE, the vertical diffusion plane is at infinity. Chromatic dispersion occuring during white-light reconstruction makes vertical diffusion seem to occur from the plane of the stereogram, because it is "hinged" there (see Figure 4.15a). The same situation is diagrammed in Figure 4.15b for a reflection recording and playback geometry.
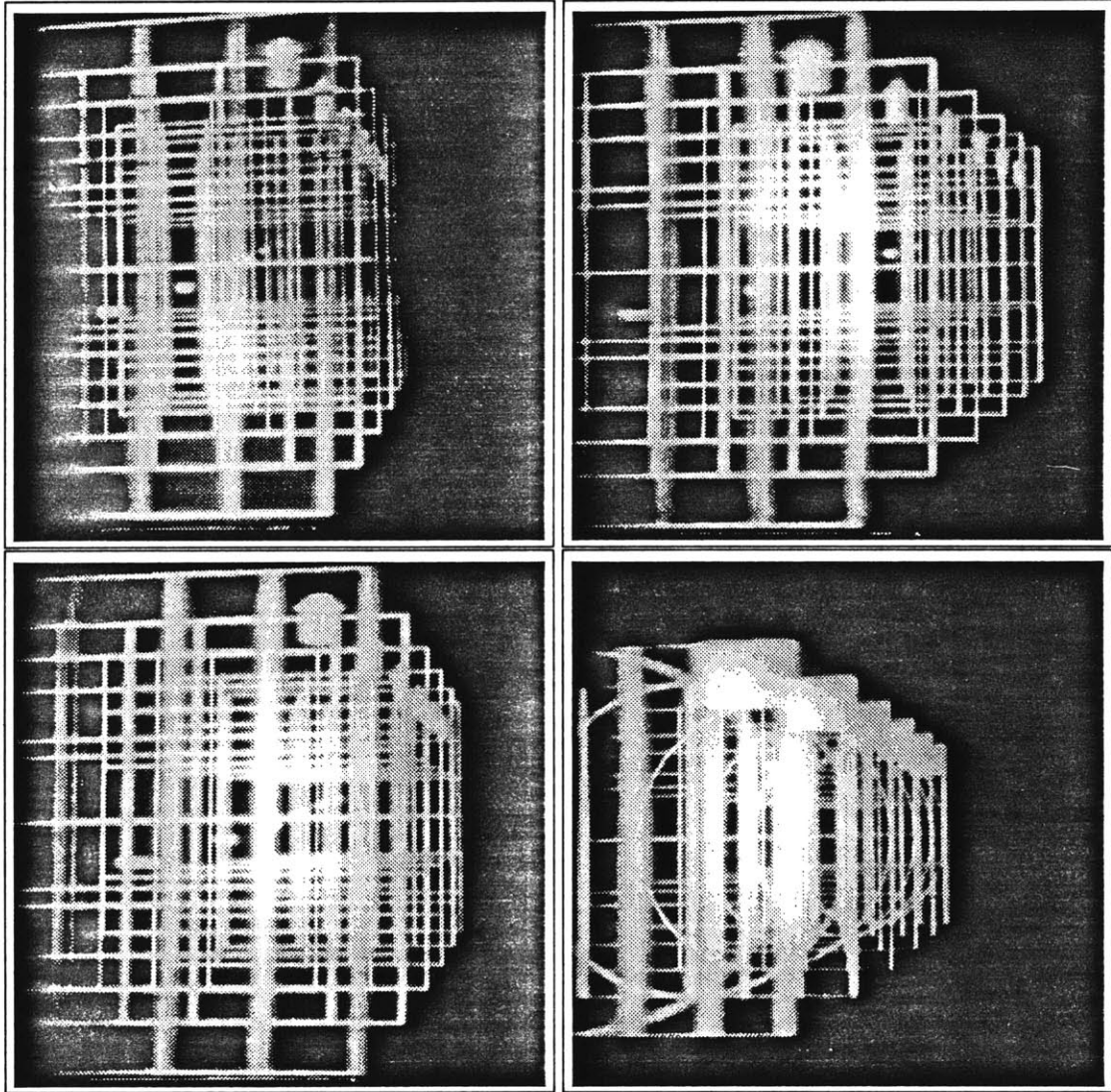
Figure 4.14: Test pattern image comparison showing aliasing artifacts and relative line widths. Top, L-R: 45 DEGREE CYL LENS HOE Ultragram, 1MM RECT PDD HOE Ultragram, bottom, L-R: 1 MM GAUSS PDD HOE Ultragram, 0.5 MM GAUSS PDD HOE Bandlimited Ultragram.
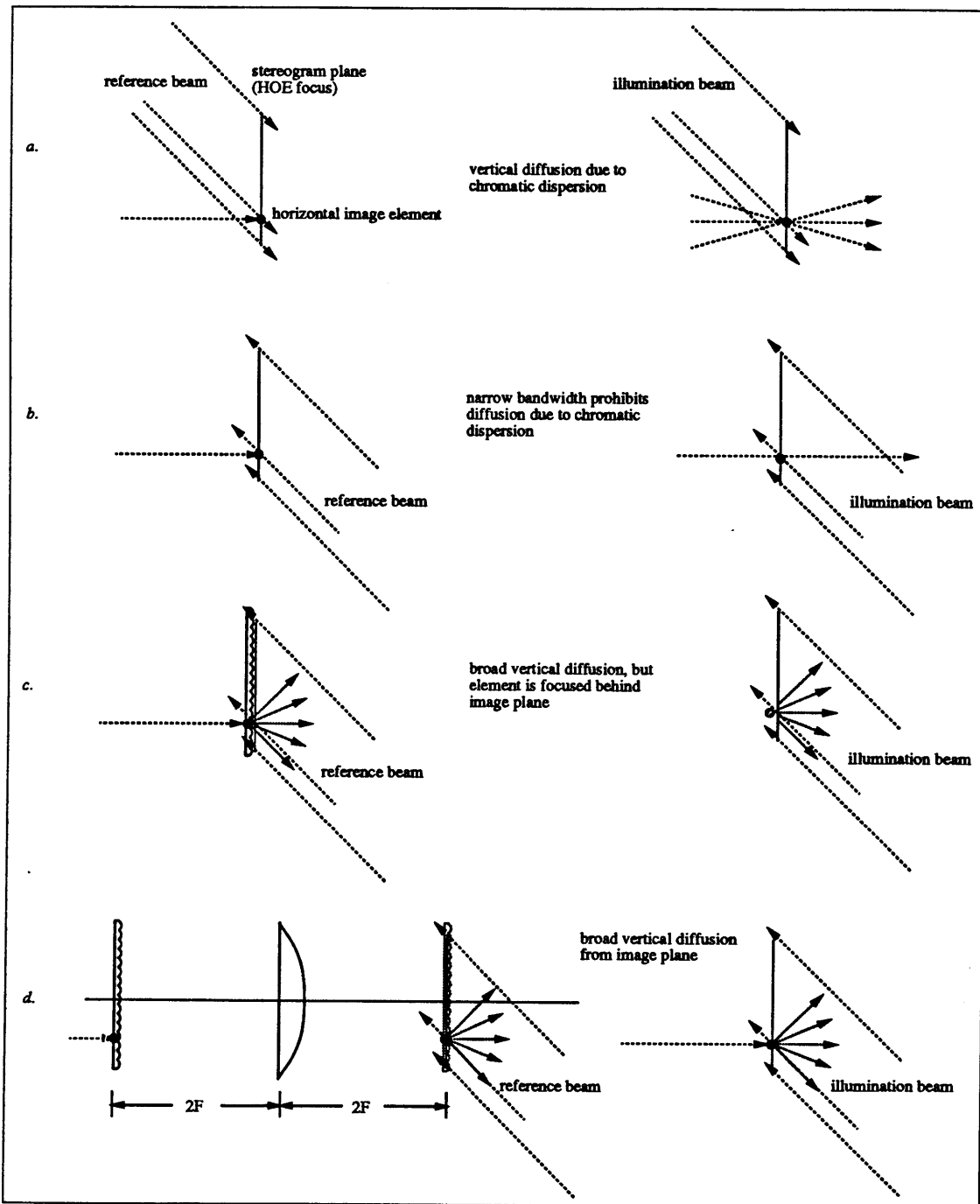
Figure 4.15: Vertical diffusion options for one-step holographic stereograms.

Placement of a vertical diffusing element, such as a lenticular screen, close to the image plane during recording as in Figure 4.15c also insures that horizontal details will focus there, but this method only works well in reflection mode because the reference beam is obstructed in the transmission version of this configuration. Finally, the vertical diffusing element can be imaged to the image-plane using a large-aperture cylindrical lens in line with the PDD HOE or CYL LENS HOE as documented in Figure 4.15d. Huff discusses all of these possibilities in [16]. At the outset, the third case seems to be both the most favorable and most difficult, allowing either achromatic transmission one-steps or volume reflection one-step stereograms to be made without obstructing the reference beam, but requiring a very large cylindrical optic in order to be physically realized. However, this optic could potentially be an HOE, and if it were sandwiched with our current PDD HOE, the system could be favorably compacted. We used a physical diffuser at the image-plane (the second method) for our experiments, however, since it met our immediate purposes, and was adequate for volume reflection exposures.

Critical issues in vertical diffusion and dispersion lie mainly in the realm of vertical view angle and bandwidth selectivity. Initial experiments for the one-step Ultragram were conducted with no vertical diffuser. Thus, due to the lack of chromatic dispersion in the volume reflection hologram, only the portion of the stereogram aligned directly with the viewer's eyes could be seen. A lenticular screen with a pitch of 64 elements/inch was then introduced into the recording system at the image-plane. This screen produced a bright, narrow band image that was viewable over a vertical angle of approximately 35 degrees. This viewangle is sufficient, however it does not match the viewangle of approximately 90 degrees achieved in the two-step Ultragram case. A lenticular screen with a 108 element/inch pitch was finally used which increased the vertical viewzone to nearly 90 degrees, but also narrowed the bandwidth and thus dimmed the image to some extent. A 1-D in-line holographic diffuser would be the ideal replacement for the lenticular. The diffusing angle of the holographic diffuser can be tuned in the recording step to give just the desired amount of dispersion in the stereogram. Experiments done to this end proved promising, but inconclusive due to the inefficiencies encountered with

the holographic diffuser.

There is an important difference between the vertical diffusion encoded in a two-step Ultragram as compared to that of a one-step. Figure 4.16 compares the two step case, in which
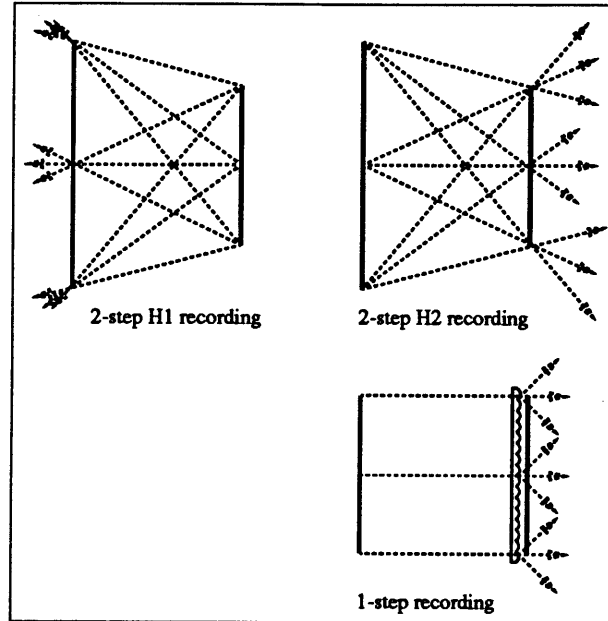
Figure 4.16: Comparison of vertical diffusion recording properties in one-step and two-step Ultragram systems.

large number of real images of a ground glass diffusing screen are simultaneously reconstructed on the transfer plane with a one-step in which each horizontal image element is imaged to a range of points on the image-plane. If we consider a single diffusing element on the ground glass surface, we can see that a focusing occurs on playback which converges light rays to the real image of the element then diverges them again. When this element's real image is placed on the transfer plane, horizontal elements are focused directly on the image-plane, and consequently the resulting high diffraction efficiency produces a very bright image. In the case of the one-step, the vertical diffusion plane is actually physically separated from the stereogram plane by approximately 8 mm, because the lenticular screen cannot be placed closer than this

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distance. When an incoming ray intersects the diffuser, it becomes dispersed over a fairly large vertical range on the stereogram surface. Thus, although vertical dispersion occurs very near the image-plane in the one-step case, the brightness of the resulting image suffers because the intensity per horizontal image element is spread instead of focused onto the image-plane. The most straightforward approach to remedying this situation is imaging the vertical diffusing element onto the stereogram plane, as detailed previously.

Because the one-step Ultragram is a horizontal-parallax-only three-dimensional imaging system, the vertical focus remains on the image-plane, and thus vertical dispersion due to chromatic bandwidth of the hologram does not significantly affect the sharpness or clarity of horizontal detail. This is not the case, however, with diffusion and chromatic dispersion in the horizontal direction.

The horizontal diffusion plane, by virtue of the 1-D PDD HOE, lies on the HOE plane. Referring to Figure 4.17, a single point projected on the HOE has its horizontal focus on that plane, and the wavefront emanating from that point can be modeled as a plane wave in the horizontal direction from the point of view of a small slit. We can calculate the chromatic dispersion of a point on the extreme edge of the HOE, which makes the largest angle with respect to the integral slit focus using a modified simple raytracing equation:

$$\frac{\lambda_2}{\lambda_1} \sin \theta_{obj} = \sin \theta_{dev},$$

where

$$\lambda_2 = 630nm,$$

$$\lambda_1 = 647nm,$$

$$\theta_{dev} = \text{output angle for wavelength farthest from recording wavelength}$$

implies a 17 nm bandwidth for this example calculation. A point projected on an f/1.0 HOE

View from above

edge point ——————— f/1.0 HOE plane

center point

edge point ——————— f/0.5 HOE plane

integral (HOE focus) plane - - - - - - - - - - - - - - - -

$\theta_{f/1} = 26.5°$ $\lambda_1 = 647\,nm$

$\theta_{f/.5} = 45°$ $\lambda_2 = 630\,nm$

$\theta_{f/1}$

$\theta_{f/.5}$

viewer

reference source position,
illumination source position

λ_1
λ_2

λ_1
λ_2

$\Delta\theta = 1.5°$

$\Delta\theta = 0.75°$
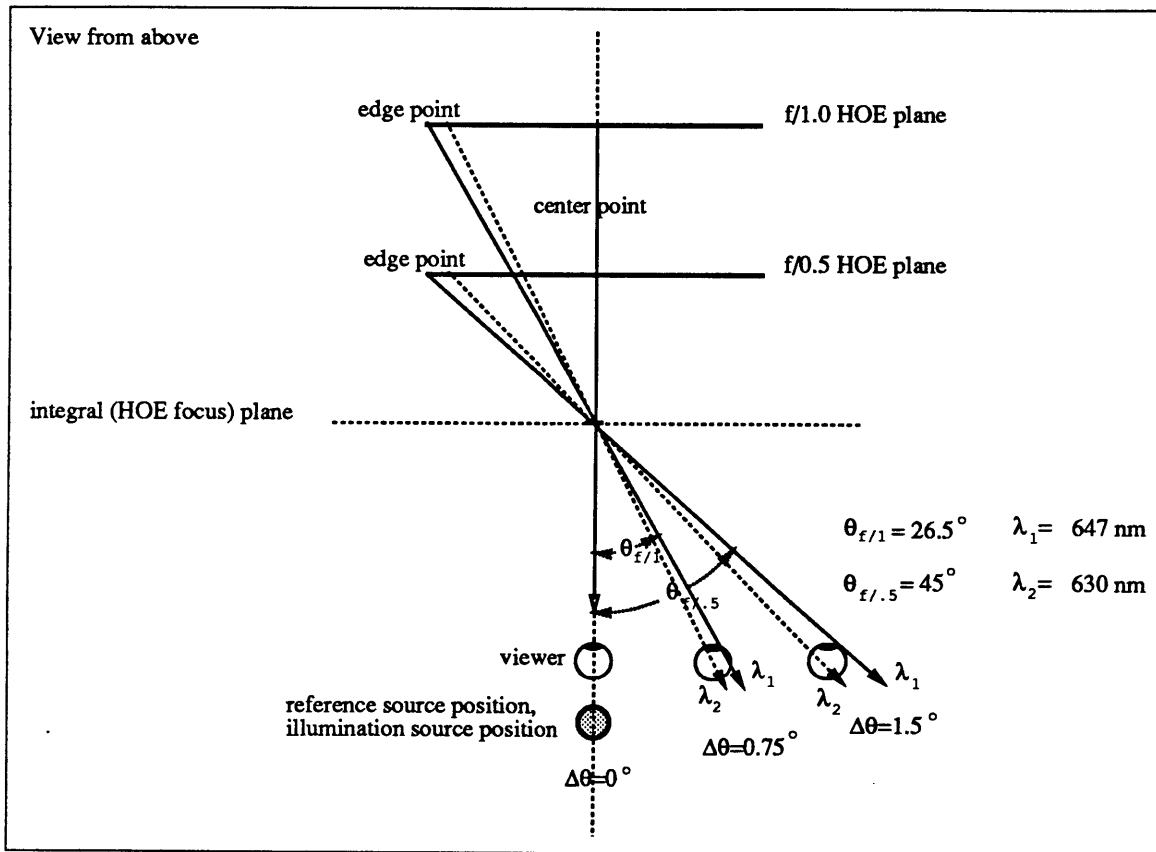
$\Delta\theta = 0°$

Figure 4.17: Comparing horizontal chromatic dispersion in an f/1.0 system and an f/0.5 system.

makes a maximum θ_{obj} of 26.5°, while an f/0.5 system makes a maximum of 45°. The dispersion angle is given by the difference between θ_{dev} and θ_{obj} due to the fact that we are using direct illumination, and λ_1 equals the recording wavelength. Our equations thus boil down to:

$$\Delta\theta = \theta_{obj} - \sin^{-1}\left(\frac{\lambda_2}{\lambda_1}\sin\theta_{obj}\right).$$

This equation and bandwidth yield a horizontal chromatic dispersion angle of 0.75° for the extreme angle in the f/1.0 case and 1.5° for the extreme angle in the f/0.5 case. It is important to note that the increased dispersion angle only affects vertical lines which are visible over the extra viewing angle width that the f/0.5 HOE affords. Thus, the horizontal chromatic dispersion of the f/1.0 system is a subset of that of the f/0.5, and the volume of the image which is viewable in both cases experiences no dispersion changes dependent on HOE speed. Of course no horizontal chromatic dispersion is experienced for points on the illumination/integral exposure axis.

In conclusion, there exists a direct relationship between HOE f-number and subsequent stereogram angle-of-view and horizontal chromatic dispersion. However, the only parts of the image that suffer are the "extra perspectives" that the faster HOE allows the viewer to see as compared with the slower one. Qualitatively speaking, the chromatic-dispersion/view angle tradeoff appears reasonable, as evident in the two-step Ultragram examples which have the same angle-of-view characteristics.

Chapter 5

Scaling Up the System

A major concern in formulating a geometry for the one-step system was *scalability*. Scalability refers to the construction of a physically larger printer to produce stereograms on the order of meter-by-meter in size. Chapter 2 describes some of the early experiments in large scale Ultragrams for CAD/CAM hardcopy applications. The limitations of these techniques were mainly two-fold. One, the projection system used was very inefficient, consisting of the standard projector and a ground glass screen. Due to the isotropic nature of the screen, approximately 99% of the light projected on it was wasted, and did not contribute to the exposure at all. Consequently, exposure times were higher than desired, and reference to object beam ratios were also too high. A second limitation to the system was the isotropic scattering of the diffusion screen, a property that made playback of the stereograms in white light unfavorable due to vertical chromatic dispersion. Both of these problems have been solved in the 1-D preferred-direction diffusing HOE system described in the previous two chapters.

Early experiments with large scale Ultragrams have proven very promising. The meter-square stereogram printer documented in Chapter 2 was designed around a novel film shuttle, capable of printing very wide one-step stereograms. A schematic of the system with the shuttle

is shown in Figure 5.1. In this system, the object beam is projected onto an isotropically-diffusing coated screen, which is placed 0.5 meters away from the holographic film plane. The reference beam is directed to the slit via a novel traveling source tower, which serves to change the side-to-side reference beam angle for each exposure. In this way, a point source could be used to illuminate the final stereogram without distorting the image, since each integral would be illuminated by an approximation to the original reference beam used to record it. The experiments were conducted for transmission mode, due to its more straightforward geometry and relaxed stability requirements (as compared with those of reflection format).

The results of past experiments are very promising. The major shortcomings of the recording system are the chromatic dispersion properties associated with the isotropic diffuser, and the extreme amount of light necessary for the object beam component. Replacing the projection screen with an HOE was the obvious choice for solving both of these problems simultaneously.

HOE Recording

Special attention was given to a number of issues concerning the 1-D PDD HOE that insured ease in scalability. The MIT meter-square stereogram producing facility retains the important consideration that it must fit in a relatively small area; a constraint that echoes the desire to eventually compact the system down to a portable size. A low f-number HOE decreases the amount of space necessary to project image information into the integral slit. Also, the shallow angle chosen for the off-axis projection onto the screen fits into the size limitation plan, although a 0-degree projection would be more ideal. The meter-square system cannot require image-size-apertured refractive optics for obvious cost reasons, so we must rely on diffractive optics to move the light around. This may present a problem due to the steep illumination angles required for the large scale system because the HOE will no longer be illuminated by its phase conjugate. The fact that only a single element is used, rather than the composite element
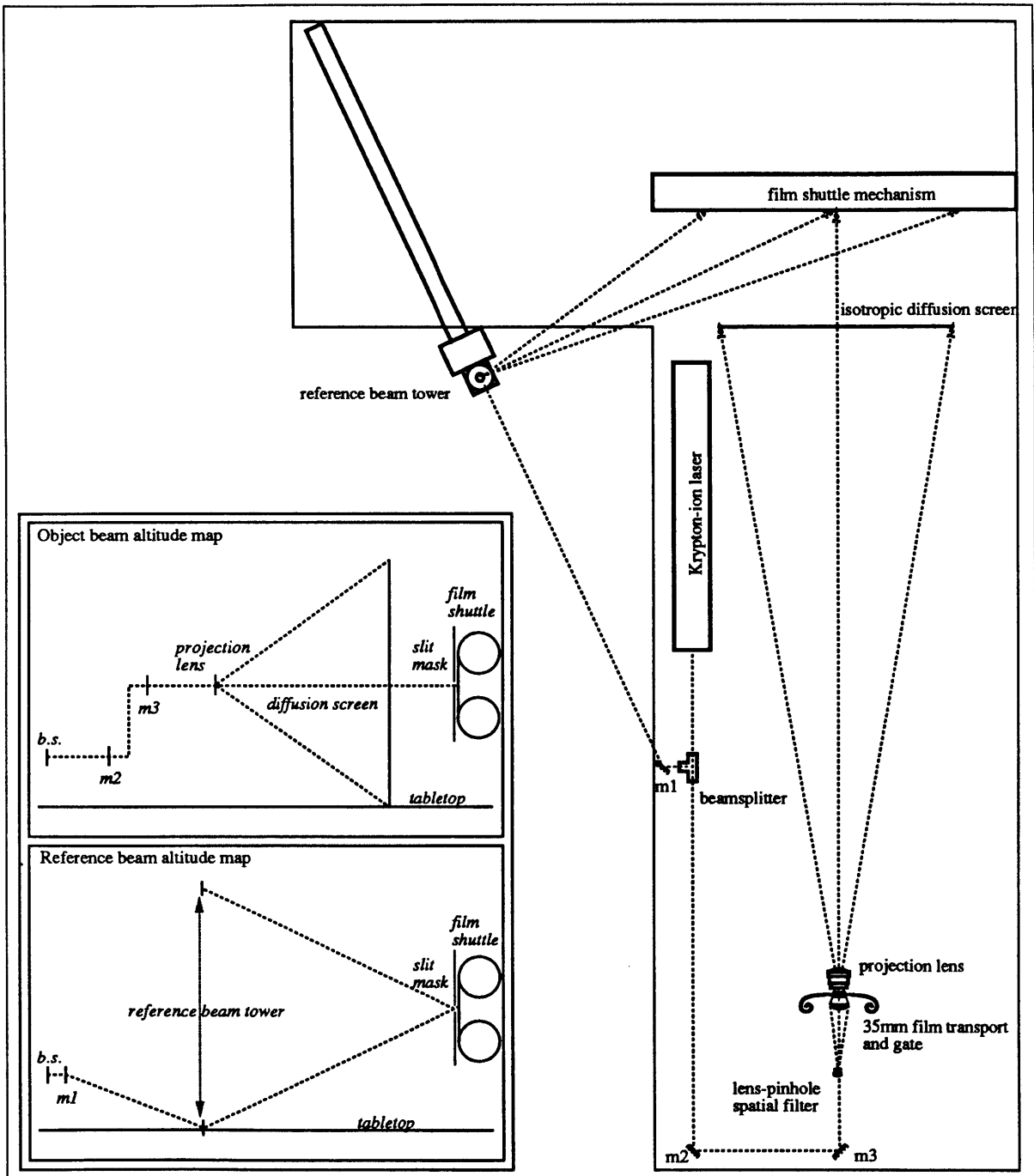
Figure 5.1: Meter-square one-step Ultragram printer schematic. (System is rotated 90 degrees from viewing geometry.)

demonstrated by Benton[4], makes it easy to produce the HOE itself. Inherent in the integral HOE scheme, of course, is the fact that the HOE recording itself does not require a large amount of space, or large hardware apart from the already-existing film transport system. Zero-order suppression can be accomplished in large scale by tiling the HOE with louver film panels like the ones used for the small scale system. Spatial limitations have thus been taken into account in designing the meter-square Ultragram system as well as recording the HOE that makes it possible.

In recording a large integral HOE we must make due without some luxuries that are available only for smaller-size configurations. For instance, collimators are unavailable in sizes necessary to expose meter-wide integrals. Although it would certainly be easier to make a diffractive slit collimator, it is still a difficult task. It is also not out of the question to have a slit reflective collimator fabricated, certainly a much less expensive proposition than a full aperture collimating mirror. Possibly the best compromise is to use a special grating, as diagrammed in Figure 5.2, which takes a diverging source as input and diffracts a slit beam which is pseudo-collimated. Such an element would be relatively simple to make, since it would not require large aperture optics, and could itself be an integral element. Still, in recording the primary tests, no collimator or grating will be used; implying a configuration that will undoubtedly cause some distortions. The setup to be used for recording the meter-square integral HOE is diagrammed in Figure 5.3. A long mirror is used to produce the 30 degree reference beam, since the film translator cannot be turned on its side for recording. The object beam incorporates a similar diverging element as the small scale. Since illumination will be in the form of a diverging beam phase-conjugate, the position of the object beam element will be moved so that upon reconstruction a focus will be formed 0.5 meters in front of the HOE. The final integral HOE film will be laminated to a piece of sheet glass and sandwiched with tiled louver film.
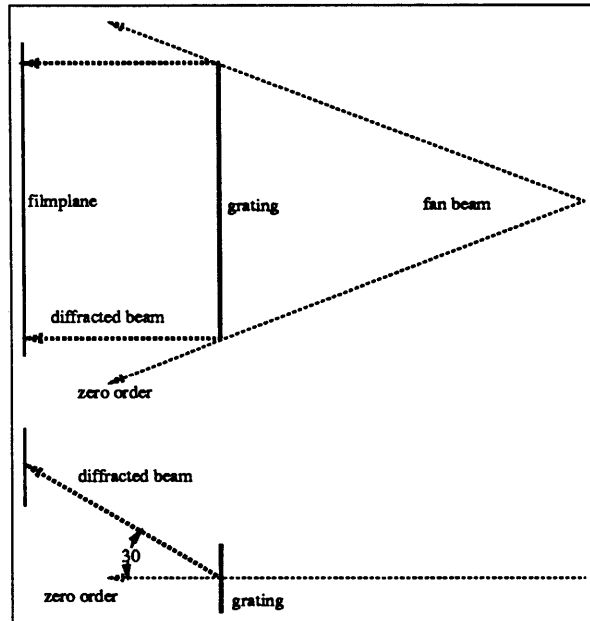
Figure 5.2: Grating for producing collimated reference beam for large scale integral HOE recording.
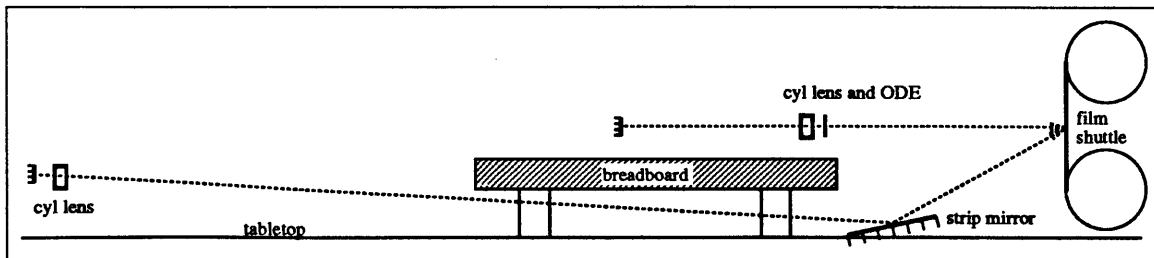

Figure 5.3: Side view of setup to be used for recording meter-square integral HOE

Large Scale Printer

The proposed large scale Ultragram printer will have a few changes compared to its earlier counterpart. Figure 5.4 documents the proposed setup. In this case, the object beam is projected from a tilted plane to the appropriately-tilted plane of the HOE. The reference beam may either be in the form of a stationary diverging fan beam or a traveling reference source, as previously described. The HOE will insure no vertical chromatic dispersion, and excellent conservation of light, while providing the same angle of view as the previous isotropic diffusing system.

A fundamental concern for projection of the object beam in the system (without a collimator) is the tilted nature of the stereogram plane. In the small scale system, the collimator served to make parallel all projected vertical lines, thus eliminating the keystone effect usually seen when projection screen normal is not parallel to the projected beam. In Figure 5.4, the film plane is tilted parallel to the HOE plane, and the condenser lens focuses the light into the center of the projection lens, thus insuring tilt correction (ideally). If this solution proves to be problematic, predistortion will be accomplished in the graphics for the projected images. One such solution would be to render the images as if they were photographed onto a 30 degree-tilted film plane. Depending on how difficult this manipulation is, this may or may not be the preferred method of distortion correction.

One of the difficulties encountered in producing very large scale reflection stereograms using a standard diffusion screen is the need for a physical slit to mask the object beam side of the film during exposure. This slit delineates the exposure boundaries, therefore it must be precisely machined and its width tuned over the meter-width of the film, a difficult task. In addition, a second slit must be introduced on the back of the film to insure that both the reference and object beams expose the same area of film. This combination of two large, cumbersome physical slit masks make it very difficult to arrange the advance mechanism, and insure adequate clearances in the meter-wide stereogram film printer. Replacement of the large diffusion screen with a holographic optical element that focuses the light down to a slit image
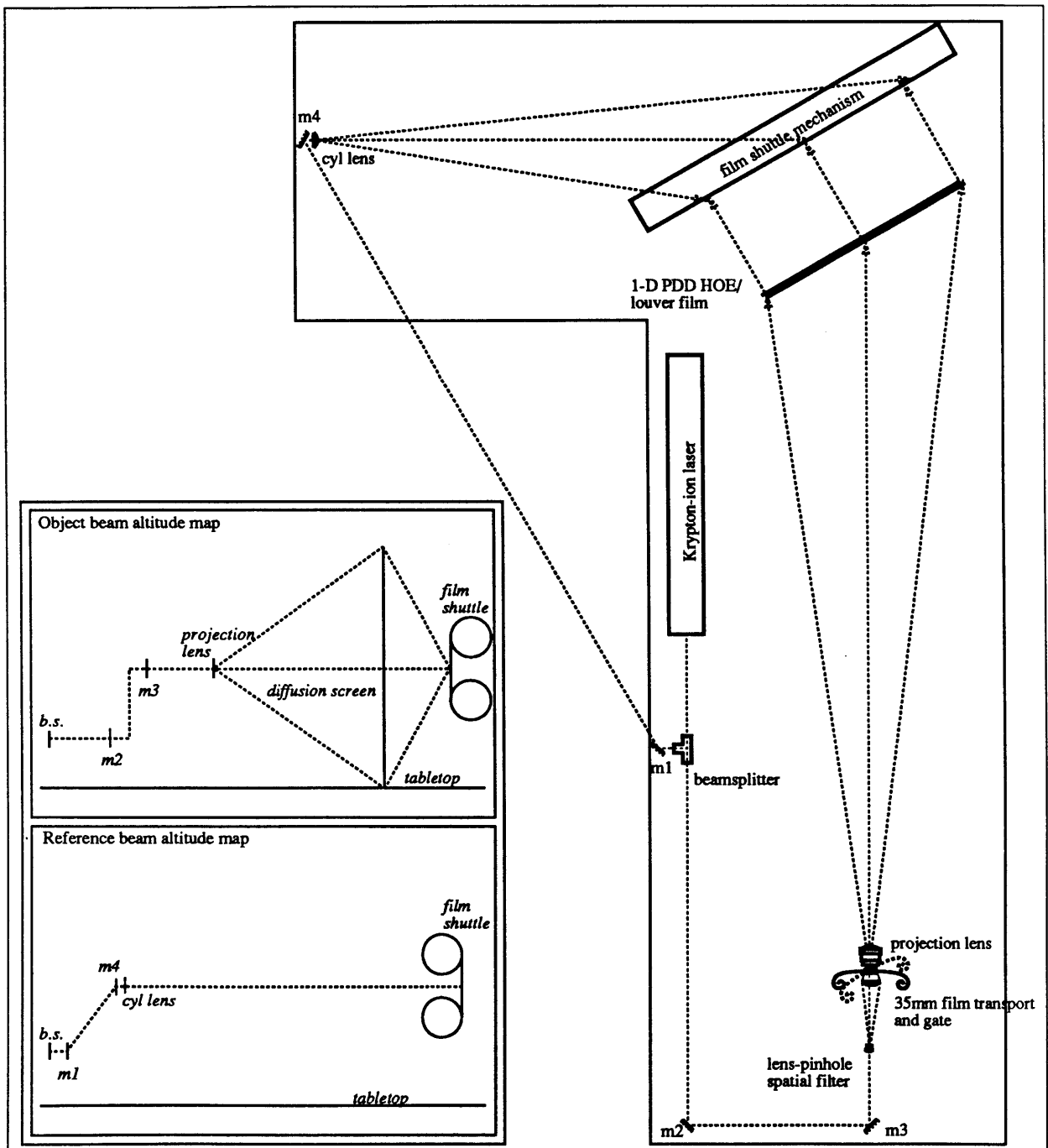
Figure 5.4: 1-D PDD HOE meter-square one-step reflection Ultragram printer schematic.

eliminates the need for a physical slit on the object beam side. In addition, if a Gaussian slit focus is produced, no slit is needed on the reference beam side either, making it possible to expose large scale stereograms without the need for long, precision-machined slit masks. In addition, exposure abutting techniques used for the small scale one-step Ultragrams described previously can be used for large scale, vastly reducing the contiguity errors experienced in past large scale exposures.

Displaying large scale one-step images may have other significant problems. In the past, film flatness has been an important issue in reflection one-step stereography[19]; the reflection configuration is much less tolerant to minor fluctuations than transmission. Also, there is the issue of mismatching the illumination beam profile with the reference beam, a practice that the scaled-down versions of the one-step seem to tolerate, but the large scale may not.

It may be possible to solve the illumination problems by incorporating a traveling reference beam, as in the transmission meter-square stereogram detailed in Chapter 2. In this case, the reference beam would again have to pass through a physical slit mask, due to inaccuracies in the beam positioning motors. The object beam could be made to fit the intensity profile of the mask by recording the HOE with a similar mask across the diverging element as in RECT FOCUS HOE. A translating reference beam would allow for near-perfect illumination with a point source positioned relatively close to the image. Unfortunately, due to the much greater sensitivity to movement in a reflection holographic recording, due to the extremely high spatial frequency of the interference pattern, a moving reference beam may not be stable enough to produce consistent results. It is possible, however, that decreased exposure times induced by the efficient HOE may improve the chances of good results.

Although no large scale Ultragrams have been created as of the writing of this thesis, the factors to insure smooth scaling of the smaller system to meter-square format are straightforward. Problems will exist, to be sure, but it is difficult to predict them with any more accuracy than has already been attempted.

Chapter 6

Conclusion

The development and incorporation of holographic optical elements into one-step stereogram printers reflects a perpetual goal of streamlining and automating the stereogram printing process. Other research goals include:

- Development of a large-scale stereogram printer that fits into a relatively small area, does not inherently contribute significant distortions to the image, and does not require an extreme amount of laser power.

- Creating a compound holographic optical element that provides horizontal diffusion at its plane, and simultaneously images the vertical diffusion plane to the stereogram image plane.

- Colorizing the one-step system by methods which may, or may not intrinsically involve the HOE component.

- Investigation of alternate recording materials such as photopolymer films.

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Some of these goals constitute straightforward manipulations of the current system, while others are intensive research topics of their own. They all represent advances which will elaborate on the preferred-direction diffusing HOE, making use of its special properties to produce one-step stereographic images.

The HOE developed here is successful because it increases the efficiency of the projection screen, compacts the system, and allows for the removal of image plane artifacts that are psychophysical barriers to perception in three-dimensions. However, the HOE does introduce some distortion problems itself, due to the fact that it too is an approximation to an optical element in the same way that a stereogram approximates a hologram. The special projection needs for the HOE require the graphics in the optical train to be more specialized than in the more straightforward diffusion screen projection system. The off-axis requirement of the HOE, for instance, implies that the images must be shrunk in their vertical dimension, and, in some cases, keystone-predistorted, before projection. Though this may be accomplished by optical means, (just as slice-and-dice itself is an optical mapping), more computationally expensive digital image processing is used in the present system in order to simplify the object beam optics train and thus conserve light. The ideal system would require only the slice-and-dice optical predistortion mapping to be performed on the perspective images, minimizing computation time. This, however, would require on an in-line optical element whose development eluded research done here. Thus, the system compromises computation time for optical unfeasibility.

Graphics development for the Ultragram, occurring in tandem, has facilitated the HOE research by providing both a means and a justification for many of the parameters used in this research. For instance, Gaussian intensity distributions of the integral exposures are theoretically necessary for proper bandlimitation of the stereogram image, but such distributions are only easily attained with a "focusing" projection screen. Thus while testing the HOE relies on the "correctness" of the images, image processing techniques rely on the "correctness" of the optical system. Still, because image processing is a relatively expensive step in the stereogram pipeline, and because it is often linked to the resolution of the image in cases where optical

analogs would not be, it is troubling that the HOE system relies so heavily upon it. Ideally, were the HOE able to work as an in-line system, the non-slice-and-dice portion of image processing necessary to project the image onto an inclined plane would be eliminated, reducing software complications and making the system more universal.

The piecewise optical construction of the integral HOE an effective means of large-aperture optic synthesization, because it enables a simpler physical scaling of the system than the monolithic method. Because one of the original premises of system development was to use a reasonably-sized laser, this applies for the construction of the HOE as well as the stereogram printer. Thus, the method of breaking the problem into literally smaller parts, has produced a viable solution. Regrettably, because the integral system is, itself, one-step, the anti-Gaussian filtration method used to create more Lambertian monolithic preferred-direction diffusers cannot be applied. Still, the integral PDDs have better center-to-edge falloff ratios than their uncorrected monolithic counterparts, somewhat vindicating this weakness.

Color is another subject well-worthy of applying to the one-step system. If we can compensate for distortions due to chromatic playout angle variation, full color Ultragraphic images will be possible. Compensation could involve the projection screen HOE, depending on the method used for color creation. For instance, if three laser wavelengths were simultaneously directed to a broadband HOE, all three could possibly be converged to a panchromatically-sensitive film for integral exposure. In this way, a chromatically-predistorted individual images, encoded chromatically on the 35 mm frames, could be projected and imaged simultaneously. If a single laser wavelength were used for separate red, green and blue stereogram film separations, the images could be properly distorted in "real time" in the printer, by translating the projection lens. In the case of a special stereogram recording emulsion, the HOE could produce three very closely-spaced foci, one for each wavelength, as in a color CRT screen. There is a great deal of room for experimentation in the color regime, and, since color is an important psychological depth cue, ideas such as these may soon be applied.

A good deal of experimental and developmental work must still be done on the large scale version of the HOE-based printer. Removal of collimating abilities in both the creating and replay of the optical elements may prove more damaging to the final image than initially predicted. It is possible that the optical effects of recording and playout with diverging beams can be nullified and corrected using a piecewise approach similar to the traveling reference beam approach used for early large scale experiments. This is an area that requires further investigation. More experimentation must also be dedicated to replay of one-step HOE-aided stereograms with phase-conjugate illumination, to allow for the plane of maximum angular multiplexing to be closer to the viewer. It is possible that changes in the printer system reference beam may aid in this endeavor. Finally, a return to the cylindrical display format is also warranted, due to the increased viewangle and image tangibility associated with that type of display. This may require changes in the HOE in regard to optical power, due to the extreme depths of the images involved. Still, since the effect of displays like the Alcove have been unparalleled, these formats should be reconsidered.

The quest for the most realistic, palpable three dimensional image continues to present substantial challenges to barriers once thought fundamentally insurmountable. The hologram, thought of as the ultimate imaging medium due to its unparalleled resolution, has evinced speculation that perhaps we don't need quite that much resolution in order to perceive objects as tangible. Indeed, clarity and resolution in the human eye are issues which are complicated by the existence of phenomena such as scattering, haze, chromatic variations, reflection, and diffraction. Finding shortcuts to mimic the *effect* of these properties on our perception without having to reproduce the phenomena themselves is a practice that lies in both the realms of computer graphics and holographic stereography.

The science of computer graphics has had a bit of a head start in the field of reproducing reality. Mathematical models of light paths and object shapes have been developed which serve well to represent objects as a photograph does – in two dimensions. Researchers in that field operate within the strict limitations of their hardcopy display, a photographic emulsion whose

properties have been studied, manipulated and tuned over the last hundred years. The process of synthesizing three dimensional object images on the other hand, is much more arduous because of the lack of a perfected medium and a method to record onto that medium. The sheer volume of information needed to convey a sense of three-dimensionality to the viewer has induced the development of horizontal-parallax-only displays, and a coarse sampling of perspective views to be encoded in the display device. By intelligently utilizing the continuum of information which can be modeled from the physical world, the three-dimensional display of information using a holographic stereogram medium can adequately promote a notion of palpability and reality. Artifacts within the medium, however, can serve to disturb or even destroy the image palpability, at which point the image becomes degraded to the level of curiosity and gimmickry.

Coexisting with the limitations of the medium is the desire for high speed, simple production. The automation of a system allows it to be more universally accepted, and configures it as a "black box", a general tool, to be used in further experimentation and application. Simplicity makes the system available to a wider spectrum of individuals, requiring little or no special instruction for operation. Such a system can then be expanded, perhaps to make larger output, as the case may be. Thus, it is only with the automation and simplification of a system that that system can become indispensable.

These are the two issues that confront the evolution of one-step holographic stereogram printers: to keep the output within the bounds of perception of tangibility, given the limitations of the medium, and to simultaneously be simple and automatic, in order to promote usage and become a tool for visualization. Separately, these two issues are far from being fully realized; however, the struggle with both together implies a very difficult task. Interestingly, the two problems do not always grind against one another, however, as is apparent in the work done here. The evolution of HOEs for stereogram printers, motivated mainly by the second issue of moving toward a more automated system, has enabled the demonstration of integral image bandlimiting, which was impracticable with past systems. This, in turn, makes it possible to finally chart the limitations of the stereogram technology and map specific areas of concentration

which would improve the medium. The work done here thus represents a necessary step to charting the direction of future research in the field of holographic stereography.

The existence of a configurable holographic optical element in the imaging system of a holographic stereogram provides us with the ability to use alternate recording materials, such as photopolymer films, which could not be used previously due to their power requirements. The ease in processing associated with these materials can then be taken advantage of in printer automation and design. We can also speculate on the use of an achromatic HOE system for the production of full color one-step images using a panchromatic recording material and three laser wavelengths. The physical size expansion is a topic which has been addressed at length in this thesis, and which could include alternate formats, such as curved Alcove-like one-step images.

The HOE technology itself can also be developed. The novelty of the integral recording system can enable the construction of very specialized projection screens for use with curved format or for achromatic imaging purposes. Alternate diffusion properties must be investigated in order to optimize the efficiency and Lambertian qualities of the HOE screen. Finally, the incorporation of an in-line system, if it is at all possible, constitutes an ideal goal.

If the HOE system, and the stereogram printer of which it is an integral part, can produce acceptably palpable representations of reality, then automation and simplification can proceed. If the evolution of the one-step printer goes the way of the photocopier, then perhaps the medium will improve as the need for output increases. At this point it is difficult to speculate; however, the desire to experience a palpable representation of reality motivates hope in the full development of the three-dimensional hardcopy machine.

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