Multiplex holography: some new methods

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Abstract. Afocal cylindrical lens systems and their prismatic equivalents are used in the formation of multiplex holograms. They introduce the desired, controlled astigmatism, and they assist the low f number final cylindrical lens in the formation of a strip hologram. In addition, a line source and a broad source fringe system applied to the hologram-making process can result in improved signal-to-noise ratios.

Subject terms: multiplex holograms; cylindrical holograms; holography; anamorphic systems; broad source interferometry.

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1. INTRODUCTION

In the multiplex hologram process, a hologram is synthesized from a multiplicity of photographs, the photographs being taken of an object from different directions along a horizontal arc (Fig. 1). To synthesize these pictures into a hologram, each picture (typically a 16 or 35 mm motion picture frame) is reimaged through a projection system illuminated with a laser. A cylindrical lens at the image plane then causes rays from all parts of the image to pass through a vertical slit, and a hologram is formed by bringing a reference beam in from either above or below (Fig. 2). In this way, N frames are converted into N long thin holograms that together constitute a composite hologram. As developed by Lloyd Cross, if the reference beam is introduced from the top or bottom, the hologram is white-light viewable for exactly the same reason that the Benton, or rainbow, hologram is white-light viewable.

The most difficult problem in such holography is obtaining a suitable cylindrical lens. The lens should be of low f number, preferably f/1 or better. Such lenses are either expensive or difficult to procure, or both. A single-element cylindrical lens of low f number suffers from severe aberration, principally spherical aberration, or rather, the cylindrical equivalent. Cross and associates used a lens formed from a Plexiglas-constructed lens-shaped cavity filled with mineral oil.[1] Such a lens is tuned by means of clamps at various positions along the edge, and the aberrations can thus be empirically reduced to an acceptable level. Alternatively, Fusek and Huff[2] have made diffractive, or holographic, lenses formed by interference between a plane and a cylindrical wave. They have also made use of cylindrical lenses for forming the image so that the vertical and horizontal structure is imaged independently, with independent magnification for the two dimensions. They can thus form an image that is of the required length, but is narrower and can therefore be passed through the final cylindrical lens with less aberration. The proper aspect ratio is restored in the holographic image by increasing the radius of arc into which the hologram is bent in the viewing process.

In addition, it has been found advantageous to introduce some controlled astigmatism in the formation of the images from the motion picture films so that whereas the image focuses horizontally on the cylindrical lens L_{cy3}, it focuses vertically on the hologram-recording film.[2-4] The justification for this arrangement is that the multiplex hologram, like the Benton hologram, is a hologram only in the horizontal direction; in the vertical the imaging process is a conventional one.

The ideally designed multiplex hologram is thus produced by a complex imaging process, which in reality is four distinct imaging processes integrated into one.

In the horizontal direction, the system images each film frame into the vicinity of the cylindrical lens. The same system images the source to a position near the recording slit; this imaging process is accomplished in part by lens L_{cy3} and is an alternative way of describing the directing of the rays into the recording slit.

For the vertical direction, the optical system, besides imaging the film frames onto the recording slit, should image the source to a position downstream from the recording slit so that when the resulting hologram is read out, the source image, vertically, will be focused at the plane of the observer. This procedure removes the rainbow effect, the vertical variation of color, from the holographic image.

We subsequently refer to these different images, respectively, as the horizontal image, the horizontal source image, the vertical image, and the vertical source image.

Ways have been sought to carry out these four distinct imaging processes in an optimum way. For example, the additional cylindrical lenses to replace L_1 and L_2, as utilized by Fusek and Huff and by Haines,[3] effectively decouple the vertical imaging processes from the horizontal ones so that they can be treated as essentially different systems and thereby adjusted independently.

We propose here some alternative systems utilizing cylindrical lenses to achieve the same results, but in a different way. Our methods use conventional spherical lenses to perform the basic imaging processes, while using an afocal cylindrical lens system, or alternatively, a prism equivalent, to introduce the perturbations needed to achieve the required anamorphic characteristics. The systems are adaptations of methods that have previously been developed to a high perfection in the wide screen Cinemascope process.[5]

The advantages of the proposed systems are severalfold. First, the principal imaging is done by means of the more conventional, more easily attainable, and generally better developed spherical lens, with the anamorphic elements performing essentially a perturbation function. Second, all lenses are used in their normal way, imaging from the focal plane to infinity or vice versa. Third, such systems have been widely explored in previous applications and are thus relatively well developed. Finally, the second of the two proposed methods, the prism method, has enormous flexibility.

2. AFOCAL SYSTEM

Our aim is to alter the aspect ratio of the image falling on the

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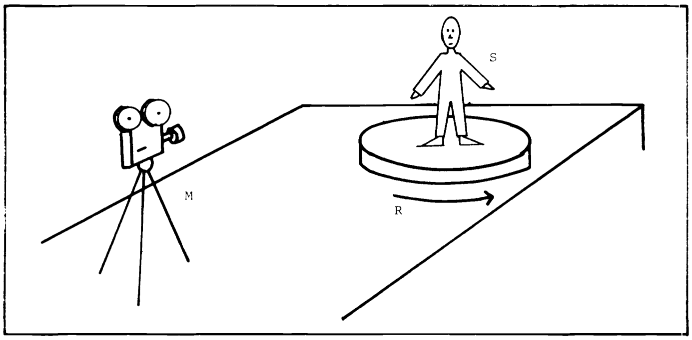

Fig. 1. Setup for recording film for multiplex hologram. S, subject; R, rotating platform; M, movie camera.

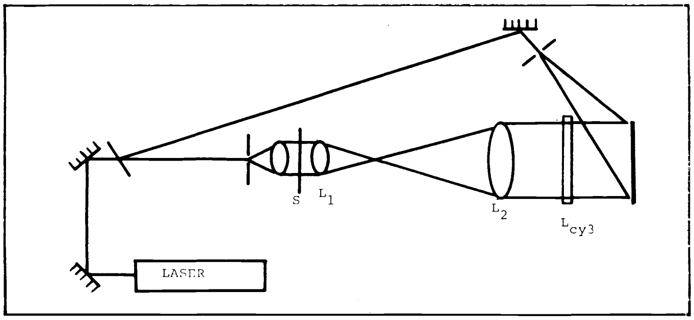

Fig. 2. Simplified diagram of optical system used for making multiplex holograms.

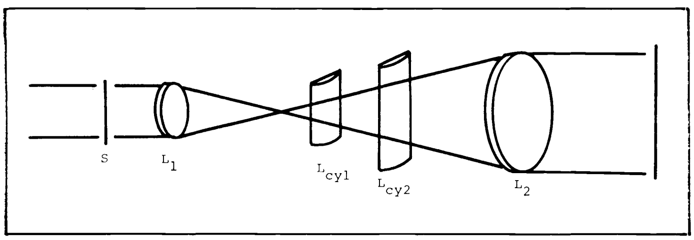

Fig. 3. Diagram illustrating lens positions when auxiliary cylindrical lenses are used.

cylindrical lens L_{cy3} so that a smaller horizontal-aperture lens, with therefore less aberration, can be used for L_{cy3}. The use of two additional cylindrical lenses in the afocal or telescopic configuration can achieve the desired result.

The principle is shown in Fig. 3. A two-lens system (just the lenses L_1 and L_2 previously shown in Fig. 2) images a transparency s(x, y) into a magnified image. The two lenses L_1 and L_2 are separated by the sum of their focal lengths (the afocal configuration: a plane wave at L_1 is reproduced as an expanded plane wave emerging from L_2). The object s is at the focal plane of L_1 and therefore is at infinity as seen from a position between L_1 and L_2. The magnification is the ratio of focal lengths, F_2/F_1.

Suppose a second pair of lenses, of cylindrical type, to be placed between L_1 and L_2 and also in the afocal configuration. The image s is seen by this lens pair to be at infinity, and the system reimages s at infinity. The magnification is just the ratio F_{cy2}/F_{cy1} of the focal lengths. The image of s thus is formed, as before, at the focal plane of L_2. The magnification in the horizontal, or x, direction is now $M_x = (F_2/F_1)(F_{cy2}/F_{cy1})$, whereas in the vertical, or y, direction it remains F_2/F_1. Thus, the aspect ratio of the image has changed, but no astigmatism has been introduced; the image is sharply focused at the same position in both the x and y directions.

By making F_{cy2} smaller than F_{cy1}, the image is compressed in the horizontal direction and can pass through the cylindrical lens L_{cy3} with lesser aberration. The image thus recorded has an incorrect aspect ratio, but the final image, as seen by the viewer of the hologram, can still have the correct aspect ratio, as previously noted by Fusek and Huff and by Haines.

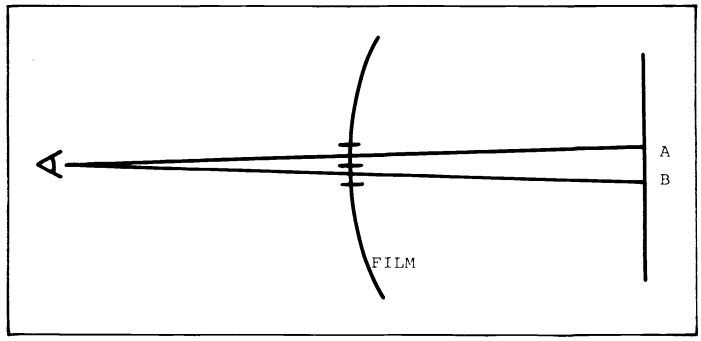

Fig. 4. Diagram showing how the image is restored to a width giving the proper aspect ratio. A and B are adjacent pixels.

We restate their reasoning as follows. The composite hologram is viewed after being bent into an arc whose proper diameter is determined by the hologram construction parameters. A viewer looking at the holograms will see, for each y = constant position, of the order of one image pixel element for each individual strip hologram. The spacing between adjacent pixel elements is related to the arc into which the hologram is bent; thus, restoration of the image to a width giving the proper aspect ratio is accomplished by bending the hologram into an arc of greater radius (Fig. 4).

The total effect is more complex since this alteration may require, for optimum design, the alteration of other parameters. With a fixed cylindrical lens L_{cy3}, this aspect ratio alteration was found experimentally to be beneficial, resulting in improved images.

A small perturbation of this system can introduce the astigmatism needed for the system optimization we noted earlier. We can either perturb L_{cy1} and L_{cy2} from their afocal configuration or move s so that it no longer lies at the front focal plane of L_1. The latter method appears the more attractive; no lenses are moved, and both lens sets remain in the afocal configuration. The astigmatism is continuously variable, the image for each dimension can be placed at the desired locations.

But recall that this imaging system has a second task; besides imaging s, it must also image the source in the manner previously noted.

With respect to the imaging of the source, the afocal cylindrical system exerts a considerable effect since the source image as seen by L_{cy1} is far from infinity. For the y dimension, the afocal cylindrical system has no effect. For the x dimension, the image is shifted. Of particular importance is the manner in which this system alters the separation between the image of s and the source image. Ideally, it would be desirable to decrease this distance so that the angle of the impinging beam remains the same, since this determines the angular size of the holographic image. The problem is not a simple one; it involves the analysis of two afocal systems, not in tandem, but one imbedded within the other, and the use of the system for simultaneously imaging two separate planes, one (the object plane) lying at infinity for the outer and the other (the source plane) lying at infinity for the inner (Fig. 5).

3. PRISMS

Alternatively, prisms can be used in place of the two afocally positioned cylindrical lenses. Although the use of cylindrical components seems to be a more direct approach, high quality prisms are far less expensive, are easier to obtain, and offer greater flexibility.

Two prisms can be used to introduce a change in aspect ratio of the image,[4,6] as shown in Fig. 6. The two lenses L_1 and L_2 are in the afocal configuration, and the object s, positioned at the focal plane of L_1, is seen at infinity from a position between L_1 and L_2. The prisms are placed in a position between L_1 and L_2. In the vertical direction, the object magnification will remain F_2/F_1. However, in the horizontal direction the magnification is now $M_x = (F_2/F_1)(\sigma)$, where σ is a

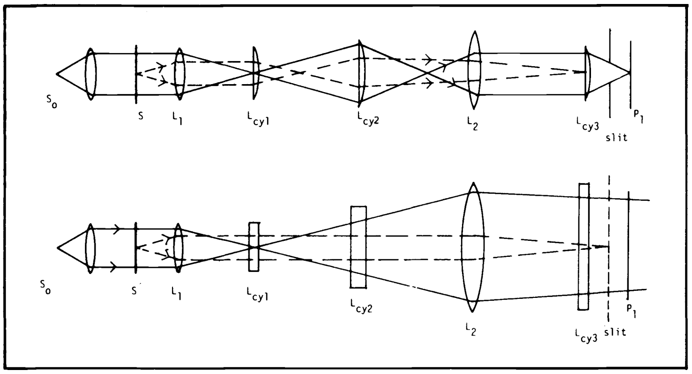

Fig. 5. Top and side view of system employing cylindrical lenses. Top: x-direction; bottom: y-direction. Solid lines show source image rays; dashed lines depict object (S) imaging behavior of the system.

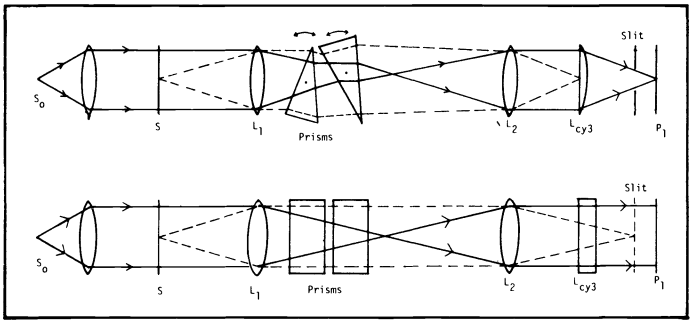

Fig. 6. Top and side view of system employing prisms. Top: x-direction; bottom: y-direction. Solid lines show source image rays; dashed lines depict object (S) imaging behavior of the system.

scaling factor introduced by the prism pair. Thus, the aspect ratio of the image has been altered without the introduction of astigmatism.

Performing a similar function as lenses L_{cy1} and L_{cy2}, the prism pair can indeed be thought of as a variable focal length cylindrical lens. The added advantage is that a continuous range of compression (or expansion) ratios can be obtained by rotating the prisms in opposite directions by various amounts. (In the previous system, the focal lengths of the auxiliary cylindrical lenses fix the amount of compression.) In effect, the prisms provide the ability to selectively determine the amount of the low f number cylindrical lens L_{cy3} to be used. In addition, as with the afocal cylindrical system, a continuously variable astigmatism can be introduced by moving s so that it no longer is at the focal plane of L_1. Thus, the vertical and horizontal image components can be focused at the desired planes.

Again, the optical system must appropriately image the source. As shown in Fig. 6, the horizontal source image should form at a plane P_1 in the vicinity of the slit (i.e., a vertical line of light should be formed). The position for the vertical source image is flexible. To eliminate the rainbow effect, we require this image to form at the viewer's position when the hologram is viewed, but this final location is affected not only by its location in the hologram-making system, but also by the divergence of the reference beam.

Since a plane wave incident on a prism at normal incidence emerges from the second prism as a plane wave, it is apparent that phase is retained. Also, since each prism individually converts a plane wave into a plane wave in a different direction, it follows that the operations performed by the two prisms individually and, therefore, by both prisms together are space invariant in the sense that the expansion or contraction of the wave along x is the same at all parts of the wave; i.e., $x \rightarrow \sigma x$, where σ is a constant describing the expansion.

A cylindrical wave impinging on a plane gives the phase distribution

$$H = \exp\left[j\,\frac{2\pi}{\lambda}\,(z^2 + x^2)^{1/2}\right] , \qquad (1)$$

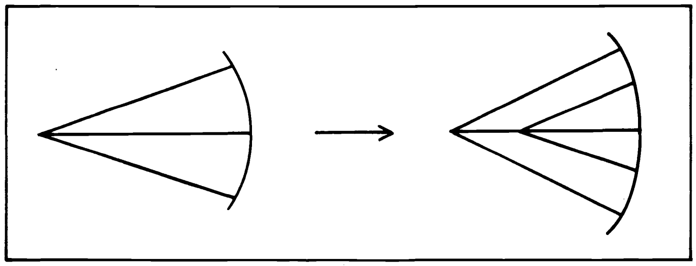
Fig. 7. Ray diagram depicting expansion of the wavefront as it passes through the prisms.

where z is the distance from the plane where the phase is observed to the plane where the center of curvature falls. Considering the plane just beyond the second prism and letting $x \rightarrow \sigma x$ yields

$$H = \exp\left[j\,\frac{2\pi}{\lambda}\,(z^2 + \sigma^2 x^2)^{1/2}\right] . \qquad (2)$$

If $\sigma > 1$, the beam is compressed. Conversely, if $\sigma < 1$, the beam is expanded.

Simple ray diagrams suggest that if the contraction is sufficiently great, the outer regions will have a shorter center of curvature than the inner regions; or, if the beam is expanded, the radius of curvature will be greater for the outer regions (Fig. 7). This flexibility suggests that it may be possible that the curvature introduced by the prisms can compensate for the spherical (or cylindrical) aberration of the final low f number cylindrical lens L_{cy3}, and the continuous variation afforded by the prisms should allow for optimum compensation.

4. EXPERIMENTAL RESULTS

Initially we made multiplex holograms without using the image compressing techniques just described. An afocal spherical lens configuration was used to magnify the transparency, and a single-element cylindrical lens of very low quality was used to condense the image to a line. We considered the question of the optimum plane for forming the x-dimension image. The basic theory suggests forming it on the surface of the cylindrical lens L_{cy3}, since then aberrations of the lens do not affect the image quality. As is generally known, this image position is not critical. Forming it at a distance equal to the cylindrical lens-recording slit distance, but on the opposite side of the slit, has the advantage that one can then view the image at its plane of focus and *after* the light has passed through the slit; i.e., one will see the image just as the hologram ideally will later produce it, and the effect of the slit on the image can then be seen prior to forming the hologram. If, however, this is done, the proper image to read out the hologram will be the conjugate one, as we now show.

The resulting multiplex hologram can be read out to form either the true image or the conjugate one. There are three considerations in making the choice: (a) The image should come to focus somewhere between the hologram surface and the position of the cylindrical lens for reasons of suitable eye accommodation, as well as for other reasons. (b) The image should have a minimum rainbow effect at the normal view distance; i.e., the image at a given viewing position should be of the same color top to bottom. (c) The image should be orthoscopic, not pseudoscopic.

If the x-dimensional image is formed at the cylindrical lens, then the source image (vertical) should form on the other side of the recording plate, typically at a distance of 10 to 50 cm. Then, in the readout process, if the readout beam is, as is usually the case, diverging (or perhaps even if it is collimated), the true image will typically form as a virtual image, and the vertical source image will form as a real image. If the observer is at the source image plane, the rainbow effect will be absent.

On the other hand, if the image is formed downstream from the recording plate, one would generally want the vertical source image to form upstream from the recording plate. One should then read out the conjugate image from the hologram in order that the image be virtual and that the vertical source image form at the observer's position.

The individual holograms that constitute the multiplex hologram form images that are only two-dimensional and are therefore neither orthoscopic nor pseudoscopic. However, these effects become manifest when the holograms are placed together to form the multiplex hologram. To form an orthoscopic true image, the images should be formed into a multiplex hologram in an order corresponding to how they were taken. If done in reverse order, the perceived image will be pseudoscopic.

Hence, to avoid a pseudoscopic image when the hologram conjugate is to be used, the holograms should be recorded in reverse order, which can be done, for example, by running the hologram recording film in the opposite direction.

The poor quality of the cylindrical lens was manifested in various ways, depending on the system adjustment. Typically, the spherical aberration caused rays from certain portions of the image, usually the center portion, to miss the slit, thereby producing a pair of black bands in the image formed by the hologram. At other slit positions, the dark bands would appear at the edges of the image; i.e., the outer portion of the image was lost. Opening the slit wider allowed all the rays to pass through, but then the slit width was much greater than the hologram displacement between successive exposures, and consequently, many holograms overlapped, giving reduced diffraction efficiency.

At the rectangle of least confusion (the cylindrical equivalent of the circle of least confusion) the slit width, although still too wide, could be narrower than when placed at other positions. However, the aberrations produced yet another defect; the object beam became strongly banded, and the bright bands were stronger than the reference beam, thus producing regions of overmodulation of the hologram and therefore yet another type of banding in the image. Increasing the reference beam-object beam ratio resulted in the reference beam being too strong overall, thus given low diffraction efficiency and poor signal-to-noise ratio.

Holograms made by the image compression techniques previously described were free from all the above problems. Both the afocal cylindrical lens system and the prism methods were tried; both gave excellent results, with the prism method being preferred because of its flexibility, which allowed us to experimentally find the optimum compression.

The prisms were positioned in the system as shown in Fig. 6, and the image was compressed so that one-half of the cylindrical lens was used. This corresponded to a compression ratio of 5:1. Now the distribution of light within the slit was very uniform, and a hologram free from defects was obtained. If the hologram were bent into an arc of the same radius of curvature that was used on the hologram made without the prisms, the viewer would perceive a compressed image. However, an image with the correct aspect ratio could readily be obtained by increasing the radius of the arc into which the hologram is bent. The image would thereby appear to focus at a greater distance behind the piece of film since the images presented to each eye become more dissimilar.

By utilizing the continuous aspect ratio change that the prism affords, we carried the process to extreme limits in order to explore the limits of the compression process. With very great compression, about 10:1, the images presented to each eye from the multiplex hologram originated from frames of very great separation, and the binocular disparity of the two images was too great for fusion into a 3-D image. In addition, some resolution was lost, which became quite noticeable at about 10:1. One possible explanation is that with such compression, the spatial frequencies of the image become finer, and the slit aperture must be larger to maintain resolution. One could readjust other system parameters to correct for these defects, but to some degree it appears that these adjustments negate the gains, although the comprehensive analysis that is needed to ascertain this is beyond our present scope.

In addition, since the image was compressed, more of the original image could pass through the cylindrical lens and be recorded.

Fig. 8. Image from multiplex hologram using prism method and 5:1 compression.

Figure 8 shows a photograph of a multiplex hologram image using the prism method to achieve a 5:1 compression. The image quality overall is comparable to the image obtained with no compression, and the horizontal information content is several times greater since more of the image was passed by the lens. Thus, it has been shown experimentally that the techniques presented offer a reasonable means of avoiding problems with the low f number cylindrical lens, specifically, by conveniently adjusting the horizontal image width at L_{cy3} so as to use just the acceptable portion of L_{cy3}, no more, no less, but still pass the entire image. However, there are also limitations to the prism method.

5. SOURCE EXTENSION

The multiplex hologram is susceptible to noise because of the relatively large number of optical elements. Indeed, care must be taken to keep these clean; otherwise the holographic image becomes objectionably noisy.

Since the multiplex hologram is a hologram with respect to the horizontal direction only and is a conventional image in the vertical, spatial coherence is required only for the horizontal direction; i.e., a vertical line light source should be entirely satisfactory, and the resulting reduction of spatial coherence in one dimension should greatly reduce noise due to scatterers on the optical elements.

There are several problems associated with the coherence reduction. First, the white-light readout capability of the multiplex hologram requires that the reference beam be introduced in the vertical plane, producing a spatial carrier term oriented along the vertical; i.e., the fringes are horizontal lines. The production of this fringe pattern requires either that coherence in the vertical dimension be retained or that the interferometric arrangement that introduces the reference beam be adjusted for broad source fringes.

Two basic interferometer types have been used to produce a broad source fringe capability in image plane holography: the grating interferometer[7] and a modified Mach-Zehnder interferometer.[8] We consider these now for the multiplex system.

In either case, rays originating from the same element of the source and traveling over the two paths, reference beam and object beam, must recombine at the hologram recording plane. In a basic way, the grating interferometer can achieve this result more completely, but the Mach-Zehnder has some advantages, such as less light loss and more flexibility.

We describe the modified Mach-Zehnder method, since the system already described (Fig. 9) is essentially a Mach-Zehnder interferometer. We now describe its modification for broad source operation. The formation of broad source fringes in a Mach-Zehnder

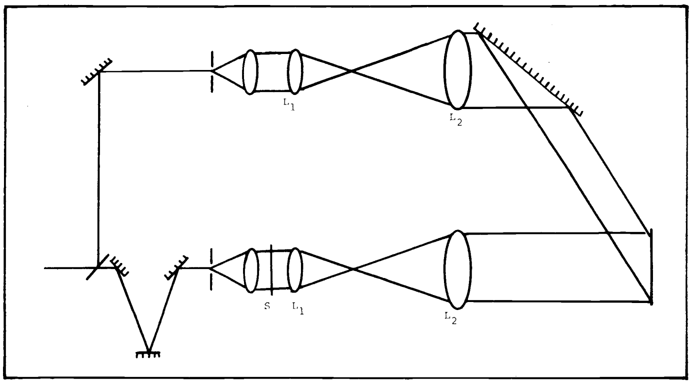

Fig. 9. Modified Mach-Zehnder interferometer.

interferometer requires the following conditions to be met. First, the two beams that combine should have the same magnification. Second, the scource images, as seen through the two interferometer branches, should be the same size. Third, the Fresnel diffraction process should be the same for both beams. To clarify this statement, the propagation of the light from the source to the recording plane can be either an imaging process or a Fresnel diffraction process. In general, it will be a mixture of both. The Fresnel diffraction process introduces a dispersion

$$H = \exp\left[-j\pi\lambda z(f_x^2 + f_y^2)\right] , \tag{3}$$

where H is the transfer function of free space and f_x and f_y are spatial frequencies. A beam traveling in the direction θ_x is said to have a spatial frequency $f_x = \sin\theta_x/\lambda$. Stated alternatively, the source image, as seen from the recording plane, should be equally distant from the recording plane via each path. We note that path equalization is required only to within the coherence length of the laser, just as in conventional laser interferometry. Finally, the two beams should each have an even number of reflections or should each have an odd number so that the source images formed by each branch have the same direction; i.e., one is not inverted relative to the other.

In view of these requirements, it is apparent that the interferometer system as constituted in Fig. 2 cannot be adjusted for broad source fringes. The modified system of Fig. 9, however, is an example of a system that can give broad source operation.

This system meets all of the above-stated requirements. The reference beam receives the same magnification as the object beam, and the two source images, as seen from the recording plane, lie at the same distance from the slit. This result has been achieved by making the optics in the reference beam essentially a duplicate of the object beam optics, although such duplication is by no means a necessary condition.

A limitation of the Mach-Zehnder interferometer is that the number of fringes obtainable is limited by the source size, being[9]

$$N = \frac{1}{2(\Delta\theta)^2} , \tag{4}$$

where $\Delta\theta$ is the angle the source subtends at the collimator, i.e., the ratio of source size to collimator focal length. This equation is essentially Eq. (41) of Bennett,[9] where for K (a conveniently small number, depending on the required fringe contrast) we have chosen 1/8. Bennett in his examples has chosen 1/10. However, N here is the total number of fringes across the field, whereas Bennett numbers his from the center outward; thus, his N will be half of that found from Eq. (4) above.

Often this fringe limitation is of little concern, since a reasonable size source (a few degrees of subtense) can result in several thousand fringes. However, the typical large angle, about 45°, between object beam and reference beam in the multiplex hologram results in about 1000 fringes/mm, or about 100,000 fringes over the entire 4 in. height of our multiplex holograms. The allowable source size is thus only a small fraction of a degree. Experience shows that even a source of such small extension can still give significant reduction of noise, particularly of the higher spatial frequency components of the noise.

On the other hand, the grating interferometer can give an absolutely unlimited number of very fine, high contrast fringes, even for a very broad source ($\Delta\theta$ 10° or more).

Another limitation, practical rather than theoretical, has to do with the depth of field of the fringes, i.e., the degree of localization. Let the source length as measured by the angle its image subtends at the recording plane be $\Delta\theta'$. It can then be readily shown that the depth Δz of the fringe localization, the longitudinal distance between the planes where the fringe visibility drops to zero, is

$$\Delta z = \frac{2d}{\Delta\theta'} , \tag{5}$$

where d is the fringe spacing.

The fringe depth should be sufficiently large that the recording slit can be kept near the center of the fringe localization region, where the fringe contrast is high. Practical problems arise. First, the slit should be aligned accurately parallel to the fringe localization plane and be precisely positioned in that plane, a feat that can be extremely difficult if the fringe depth is much less than about 0.1 mm. The inability to see directly the fringes, whose spacing is too fine for most microscopes, adds to the difficulty. Moire techniques can be used to locate the fringe plane, although these are more complicated than the direct fringe viewing that we would prefer to do. Second, the various lenses in the system may produce some curvature of field, making it impossible for the fringes to be sharply in focus over the full length of the slit.

If we choose 0.1 mm as a reasonable fringe depth, we require a source subtense not larger than about 1.2°. This limitation is about 10 times more lenient than the one imposed by the previous consideration, that of obtaining sufficient fringes; thus, if we satisfy the latter condition on source size, we will certainly satisfy the other, assuming $\Delta\theta$ and $\Delta\theta'$ are of similar size.

Finally, we suggest how to make a multiplex hologram in light that is spatially incoherent in both dimensions. We note that multiplex holography and lenticular photography are very similar processes and indeed one can be converted into the other.[10] We also note that image plane holograms that preserve phase completely can be made using spatially incoherent light.[7,11] By making an image plane hologram in a broad source interferometer, with the reference beam introduced either from above or below, the lenticular photograph becomes a multiplex hologram. The system of Ref. 7 is especially suited for such an operation for reasons given there. A lens system images the lenticular photograph into the recording plane, where the image combines with the reference beam to form a hologram.

6. ACKNOWLEDGMENTS

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